ASPECTS OF HOMOGENEITY IN THE SEMANTICS OF NATURAL LANGUAGE

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PREFACE

The topic of this dissertation is a phenomenon in natural language semantics that is known under the name of *homogeneity*. At the core of it is the fact that the sentence (1a) is true if Adam wrote all the books, but (1b) is only true if he wrote none of them.

(1) a. Adam wrote the books.
    b. Adam didn't write the books.

The work is divided into two parts. The first part, which comprises chapter 1–3, approaches a theory of homogeneous predication. The second part (chapter 4–7) explores the application of this theory to a number of linguistic phenomena. In the following, I give a short outline of the subject matter of the various chapters and their interdependencies as a guide to the reader.

It should be noted that this dissertation follows the algebraic perspective on pluralities pioneered for linguistic purposes by Link (1983), and for the most part presupposes familiarity with the relevant concepts. For an excellent introduction, cf. Champollion 2010: ch. 2.

CHAPTER 1 introduces the phenomenon of homogeneity in some empirical depth, showing that it is, in fact, wider in scope than has hitherto been clearly recognised, and argues for a particular way of conceiving of it: as a joint constraint on the positive and the negative extension of predicates. More specifically, a (potentially plural) individual in the positive extension must not have any members in common with a (plural) individual in the negative extension. It will be shown that homogeneity is not tied to distributivity, but occurs also with collective predicates, and is, indeed, a pervasive phenomenon in natural language that is not restricted to plural individuals, but can be found across a wide variety of domains.

CHAPTER 2 is highly technical and is concerned with the problem of defining a quantified trivalent logic that models homogeneity as it is found in natural language and correctly predicts its behaviour in complex sentences. This chapter is not a prerequisite for those that follow and may therefore be skipped without impinging significantly on their comprehensibility. (Prerequisites: chapter 1)

CHAPTER 3 deals with the phenomenon of *non-maximality*, which consists in the fact that plural predication in many contexts allows some exceptions. A sentence like (2), for example, can be felicitously used to describe a situation where all of ten professors smiled except Prof. Smith who is known to never smile anyway, so that his failure to smile doesn't mean much.

(2) The professors smiled.

I argue that this is a pragmatic phenomenon based on a principle that allows a speaker to use a sentence that is neither true nor false as long as the actual situation is, for current purposes, equivalent to a situation in which the sentence is strictly true. Since only sentences with a homogeneity-induced extension gap can be used non-maximally, the so-called *slack-regulating* effect of *all* then follows from its homogeneity-removing semantics. (Prerequisites: chapter 1)

CHAPTER 4 presents a theory of the exhaustivity implication in English *it*-clefts which is based on Büring & Križ 2013. The essential idea is that clefts have the logical form of copula sentences and that the identity relation is homogeneous.

(3) a. It was x that P.
b. The one(s) who P is/are x.

The exhaustivity implication of positive clefts and its disappearance under negation then follows. I also argue that clefts are not conventionally focus-sensitive and show how the effects of focus can be accounted for under the theory that is suggested. (Prerequisites: chapter 1)

CHAPTER 5 explores the idea of analysing the plurality inference of existential bare plurals as rooted in homogeneity. In the same way that (4a) is literally undefined if Adam wrote only some of the books, (4b) is undefined if Mary saw exactly one zebra.

(4) a. Adam wrote the books.
b. Mary saw zebras.

This theory is compared with previous implicature-based approaches.
(Prerequisites: chapter 1 and 3; chapter 2 is helpful)

CHAPTER 6 discusses the phenomenon of neg-raising, for which as link to homogeneity has been suggested by Gajewski (2005). A homogeneity-based theory, according to which neg-raising verbs involve not universal quantification over worlds, but homogeneous distributive predication over a plurality of worlds, is compared with Gajewski's own presupposition-based theory and Romoli's (2013) scalar implicature theory. (Prerequisites: chapter 1)

CHAPTER 7 is rather programmatic and suggests a view of conditionals, generics, and embedded questions in terms of homogeneity and non-maximality which yet awaits further formal development. (Prerequisites: chapter 1 and 3)

CONTENTS

Part I

TOWARDS A THEORY OF HOMOGENEITY

1 HOMOGENEITY: THE PHENOMENON

1.1 THE HOMOGENEITY OF PLURAL PREDICATION

1.1.1 *Homogeneity in Distributive Predication*

It is an old observation (Fodor 1970) that if a distributive predicate is not true of a plural individual, that doesn't automatically mean its negation is true of that individual.[1]

(1) a. Adam wrote the books.
TRUE iff Adam wrote all the books.
b. Adam didn't write the books.
TRUE iff Adam wrote none of the books.

There is a gap here: in situations where Adam wrote some, but not all of the books, neither sentence is true. By identifying the truth conditions of the negation of a sentence S with the falsity conditions of S, we can associate three sets of situations with every such sentence. The set of situations where it is true is called the *positive extension*, the set of situations where it is false is the *negative extension*, and the remainder is the *extension* (or *truth value*) *gap*.[2] Analogously, predicates have sets of individuals as their positive and negative extensions.

The phenomenon that distributive predicates of pluralities have an extension gap of a certain is known in the literature as *homogeneity* or *polarity*.[3] I will speak of homogeneity as a property of predicates and, derivatively, of sentences that contain such predicates and have an extension gap on account of this.[4] The

1 Strictly speaking, the biconditionals in (1) may not seem plausible. Predication involving a definite plural is known to be, in the right circumstances, somewhat tolerant of exceptions. This phenomenon is the topic of chapter 3 and will be ignored for the time being.

2 I will say that a sentence "has an extension gap" if the extension gap is non-empty. The nature of this extension gap, and how it compares to other phenomena that have been associated with a sentence's failure to be definitively true or false, are discussed in section 1.7.

3 Cf. Schwarzschild 1994, Löbner 1987, 2000, Gajewski 2005, Breheny 2005, Büring & Križ 2013, Spector 2013, and Magri 2014.

4 Note that there is another use of the term *homogeneous* as a property of predicates, which can be found in Higginbotham 1994 and Sorin-Dobrovie 2014: it means that the predicate is both divisive and cumulative.

DEFINITION 1.1.
1. A predicate P is *divisive* iff for any x, $P(x) \rightarrow \forall x' \preceq x : P(x')$.
2. A predicate P is *cumulative* iff for any x, y, $(P(x) \wedge P(y)) \rightarrow P(x \oplus y)$.

The identity of name is unfortunate, as this notion will not play any role in this dissertation. It should be pointed out that all predicates that are cumulative and divisive are also homogeneous in the sense in which I am using the term, but not the other way around. Collective predicates like *meet*, for example, are homogeneous, but arguably not divisive: it is possible for *meet* to be true of a plurality of students when it is not true of every pair of students. It may not be true, for example, motivation for the name *homogeneity* is obvious: in order for a homogeneous predicate (in the technical sense) to assign a definite truth value to a plurality, that plurality has to be homogeneous (in the non-technical sense) with respect to the predicate, i. e. the predicate has to hold of either none or all of its parts.

There is a large number of equivalent ways of stating this constraint for distributive predicates. One of them is given in (2).

DEFINITION 1.2. (Simple Homogeneity) A predicate is neither true nor false[5] of a (plural) individual a if it is true of some parts of a and false of other parts of a.

This allows one to read off the negative extension of a predicate from its positive extension: it is as large as it can be without violating homogeneity, i. e. the maximal set that has no members which overlap with a member of the positive extension.[6]

It should be noted that homogeneity, in this sense, is also found not only with lexical predicates, but also with derived distributive predicates. On the most accessible reading, (2) is true if every student received a pen and five sheets of paper, but false only if none did.

(2) The students received a pen and five sheets of paper.

In fact, all distributive predicates in natural language are homogeneous. This has resulted in some attempts to locate the source of homogeneity in the distributivity operator or otherwise connect it to distributivity (Schwarzschild 1994, Gajewski 2005). Schwarzschild assumes that distributive predicates are primitively defined for atomic individuals and have bivalent truth conditions when restricted to that set: their positive extension is a set of atomic individuals, and their negative extension is the set of atomic individuals not in the positive extension — there is no gap here. In order to apply them to pluralities, both the positive and the negative extension of are then independently closed under mereological fusion, so that for any two individuals in the positive extension, their sum is also in the new positive extension, and analogously for the negative extension. Those plural individuals who are sums that have some parts in the positive and some parts in the negative extension — the ones who violate homogeneity — are then found in neither.

To illustrate, assume a model with four individuals a, b, c, d, and a distributive predicate P which is true of the atoms a and b. Then its simple positive and negative extension is as in (3a). The result of closure under fusion is shown in (3b) (where closure under fusion is effected by the * operator). A plurality that is mixed with respect to P, such as $a \oplus c$, can be found in neither and is therefore in the extension gap.

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of those at the very opposite ends of the crowd who didn't even see each other. Other collective predicates like *perform Hamlet* are not divisive, and perhaps not even cumulative, but as will be argued in section 1.1.3, they are homogeneous as well.

5 Throughout this work, I use "undefined" as a synonym of "neither true nor false".

6 If one follows this procedure, the negative extension of a predicate is automatically closed under mereological fusion. Whether closure under fusion of the positive extension is applied before or after computing the negative extension makes no difference for the overall result here.

(3) a. $[\![P]\!]^+ = \{a, b\}$
$[\![P]\!]^- = \{c, d\}$
b. $[\![{}^*P]\!]^+ = \{a, b, a \oplus b\}$
$[\![{}^*P]\!]^- = \{c, d, c \oplus d\}$.

Gajewski, too, assumes that distributive predicates are primitively defined only for atoms and that a distributivity operator has to be applied to them in order to make them applicable to pluralities. That operator then simply adds a presupposition that the plurality is homogeneous.[7]

(4) $$[\![\text{DIST}]\!] = \lambda P.\lambda x : (\forall x' \prec_{AT} x : P(x')) \vee (\forall x' \prec_{AT} x : \neg P(x')). \\ \forall x' \prec_{AT} x : P(x')$$

I will argue shortly that such an approach is actually mistaken and that homogeneity is not inherently linked to distributivity. First, however, I would like to substantiate the claim that it is correct to think of homogeneity in terms of a truth value gap.

1.1.2 *Falsity, Negation, and Trivalence*

The fact that the truth conditions of (1a) and its syntactic negation (1b) are not complementary is not immediate proof that thinking about the phenomenon in terms of a trivalent logic, with separate positive and negative extensions, is necessary or appropriate.

One might think that perhaps definite plurals behave as positive polarity items and always take distributive scope over negation. That is quickly refuted. First of all, definite plurals do not, in fact, take distributive non-surface scope over anything (cf. Steedman 2012). (5) unambiguously requires that there are two boys such that each of them has read every book. The (weaker) reading in (5b), where *the books* takes distributive wide scope over the quantifier in subject position, is unavailable.

(5) Two boys read the books.
a. 'There are two boys such that each of them read every book.'
b. *'Every book is such that two boys read it.'

Furthermore, a definite plural with a bound pronoun is trapped in the scope of a negative quantifier. It makes no sense to attempt to give *his presents* wide scope over *no boy*, since that would free the bound variable *his*.

(6) No boy found his presents.

A syntactic derivation of the right meaning, based on the definite outscoping negation, would require *no boy* to be syntactically decomposed into *every boy*

7 I follow here the notational convention from Heim & Kratzer 1998, where $\lambda x : \phi.\alpha$ is writen for the function that maps x to α if ϕ, and otherwise incurs a presupposition failure. A further notational convention, which is followed throughout this work, is the use of $\preceq$ for the relation of individual parthood (Link 1983), $\preceq_{AT}$ for atomic parthood, and $\oplus$ for fusion/sum formation.

. . . *not*. Such a decomposition would predict intermediate scopings that don't exist. (7a) does not have a reading with the scoping in (7b), which would mean that for every student, there are two books that he didn't read.

(7) a. No student read two of the books.
  b. *every > two > not

Note that the postulation of a third truth value does not entail that human speakers necessarily perceive a difference between falsity and undefinedness. The third truth value could be a purely technical tool to capture, in a principled way, the fact that a sentence and its negation have non-complementary truth conditions. There is, however, already one intuitive datum that indicated that there is more to the third truth value. When a plural sentence is undefined, the natural answer is neither *yes* nor *no*, but *well*.

(8) Context: *Half of the professors smiled.*
  A: The professors smiled.
  B: Well / #yes / #no, half of them.

That speakers do indeed perceive the third truth value was demonstrated by Križ & Chemla (2015) in a recent experimental study. They presented speakers with an array of coloured shapes and asked them to judge either a positive or a negative sentence with a definite plural of the general form in (9).

(9) a. The [shapes] are [target colour].
  b. The [shapes] aren't [target colour].

There were three kinds of conditions: those where all the shapes were of the target colour, such that the sentence was unambiguously true, those where none of them were so that the sentence was clearly false, and those where some, but not all of the shapes were of the target colour. The latter kind of situations was expected to lie in the extension gap. The answer options available to subjects were *completely true*, *completely false*, and *neither*.

As controls, the authors used sentences with *all* instead of the definite plural:

(10) a. All the [shapes] are [target colour].
  b. Not all the [shapes] are [target colour].

Such sentences do not have an extension gap (cf. 1.2) and were used to make sure that *neither* answers did not just reflect partial truth. After all, there is a sense in which someone asserting "All the triangles are blue" can be said to be at least partially correct when a large number of them are, as intuitively reflected in the discourse in (11).[8]

(11) A: All the triangles are blue.
  B: Well, many of them, anyway.

8 Note, though, that unlike in the case of a definite plural without *all*, the answer *no* instead of *well* is of course also available.

In a large number of cases, subjects answered *neither* in those situations where the sentence is, from a theoretical perspective, expected to be neither true nor false. They did not do so for the controls, which shows that these judgments really do reflect the third truth value, and not some kind of closeness to truth.

Note further that gap judgments can be elicited from speakers even for sentences that have no natural syntactic negation, such as those with the quantifier *exactly two* in subject position.

(12) a. Exactly two boys found their presents.
b. ??Not exactly two boys found their presents.

(12b) is certainly odd as a sentence of English, but still Kriz & Chemla found a class of situations in which speakers judge (12a) to be neither completely true nor completely false. This will be discussed in section 1.5.

1.1.3 *Collective Predication and Generalised Homogeneity*

Homogeneity, as defined in definition 1.2, obviously makes no sense when applied to collective predicates. (13) cannot require every single boy to have performed the play; in fact, it doesn't even seem meaningful to say of a single boy that he has or has not performed a play with so many roles; he can only have or have not performed *in* the play.

(13) The boys are performing *Hamlet*.

But still, such sentences do have extension gaps that seem to be of the same nature as homogeneity violations with distributive predicates — a fact which has been largely neglected.[9]

I claim that the right way to think of homogeneity is as a joint constraint on the positive and the negative extension of a predicate. The intuition behind it is that homogeneity requires that in order for a predicate P to be *false* of an individual, that individual must not be tainted by P-ness in any way. Slightly more formally, no member of that individual must be involved in any P-ing.

DEFINITION 1.3. (Generalised Homogeneity) No individual in the positive extension of a predicate must overlap with an individual in its negative extension.

So if we have only three individuals a, b, and c, and P is true of $a \oplus b$, then homogeneity forbids it from being false of $a, b, a \oplus c, b \oplus c$, and $a \oplus b \oplus c$. If it is not true of those individuals, then it has to be undefined. P may, however, be false of c. For distributive predicates, this amounts to nothing new, but it predicts several kinds of situations in which a collective predicate is neither true nor false of an individual.

(14) Scenario 1: *Only a subgroup of the boys is staging the performance.*

9 A notion of homogeneity that applies to collective predicates plays a role in Büring & Križ 2013. The phenomenon was also independently brought up by Benjamin Spector (p. c.). I am not aware of any other discussion of it.

A: The boys are(n't) performing *Hamlet*.
B: Well / #Yes / #No, SOME of them are.

A's assertion of (13) isn't true in this situation, unless perhaps on some sort of team-credit reading.[10] By homogeneity, the fact that the predicate in question is true of a subplurality of all boys prevents the sum of all boys from being in its negative extension, and so the sentence is undefined. This can be seen from the fact that the only appropriate answer for B is the *well*-answer.

(15) Scenario 2: *The boys and the girls together are performing* Hamlet.
A: The boys are(n't) performing *Hamlet*.
B: Well / #Yes / #No, ALL the children are performing the play.

Again, the sentence isn't true in this scenario — to perform a play is not the same thing as performing *in* a play. But the boys are a part of an individual of which the predicate is true, so by homogeneity, it cannot be false, either.

(16) Scenario 3: *Some of the boys together with some of the girls are engaged in the performance.*
A: The boys are(n't) performing *Hamlet*.
B: Well / #Yes / #No, you can't say that. SOME of them are PARTICIPATING.

Undefinedness in Scenario 1 and 2 would also be predicted if homogeneity were stated in terms of parthood rather than overlap. However, it seems that (13) is also undefined in Scenario 3, where there is only overlap between the boys and an entity that performed *Hamlet*.

It will turn out to be useful to decompose the homogeneity constraint into three aspects corresponding to these three types of situations.

DEFINITION 1.4. A homogeneous predicate P can be false of an individual a only if

1. P isn't true of any part of a, (*Downward Homogeneity*),
2. P isn't true of any individual that a is a part of (*Upward Homogeneity*), and
3. P isn't true of any individual that merely overlaps with a (*Sideways Homogeneity*).

The negative extension of a homogeneous collective predicate can now be derived from its positive extension in just the same way as for individuals: simply choose the maximal set so that homogeneity isn't violated, that is to say, turn the *only if* in the above definition into *if and only if*.

1.1.4 *Homogeneous Relations*

Even though the examples so far did involve relations, one argument was always a singular, which allowed speaking of them as if they were just distributive one-

10 See section 3.3.3 on this.

place predicates with respect to their plural argument. The case of a transitive predicate with multiple definite plural arguments must not be ignored, however.

(17) The boys kissed the girls.

It is uncontroversial under which conditions (17) is false: it is false only if no boy kissed any girl. This can be seen from the syntactic negation: one cannot really say that A's utterance in (18) is true as long as any kissing happened.

(18) A: The boys didn't kiss the girls.
B: #Yes, but / #No, because / Well, Adam did kiss Nina.

One way to generalise homogeneity to relations is by defining parthood, and consequently overlap, for tuples of individuals.

DEFINITION 1.5. (Generalised Parthood) For any two n-tuples $\vec{x}$, $\vec{y}$, $\vec{x} \preceq \vec{y}$ iff $\forall i : x_i \preceq y_i$.

With parthood defined for tuples, the homogeneity constraint can now be applied to the positive and negative extensions of relations, which are just sets of tuples. From this definition, it follows automatically that (18) can only be true if all boys kissed all girls, i. e. that it distributes on both arguments.

(19) The boys kissed the girls.

TRUE iff every boy kissed every girl.
FALSE iff no boy kissed any girl.
UNDEFINED otherwise.

This is a rather strong requirement, and it contradicts the frequently made assumption that at least lexical relational predicates in natural language are closed under pointwise mereological fusion.[11]

DEFINITION 1.6. (Closure under Pointwise Fusion) For any relation R, its closure under pointwise fusion $^*R$ is the minimal relation such that for all a, b, c, d, if $R(a, b)$ and $R(c, d)$, then $^*R(a \oplus c, b \oplus d)$.[12]

This means that if a kissed b and c kissed d, then $a \oplus b$ kissed $c \oplus d$. Consequently, (17) is true as soon as every boy kissed a girl and every girl was kissed by a boy. This leaves in the extension gap those situations in which either a boy or a girl was involved in no (cross-gender) kissing at all.

(20) The boys kissed the girls.

TRUE iff every boy kissed a girl and every girl was kissed by a boy.
FALSE iff no boy kissed any girl.
UNDEFINED otherwise.

11 Krifka 1986, Lasersohn 1989, Schein 1993, Landman 1996, 2000, Sternefeld 1998, Beck 2000, Beck & Sauerland 2000, Kratzer 2007, Champollion 2010. Some of these works make use of event semantics to reach the same effective result.

12 Note that this means that $R(a, b)$ entails $^*R(a, b)$, since this is just the special case with $a = c$ and $b = d$.

Such a configuration is not allowed by homogeneity if definition 1.5 is used. In order to make room for it, one can alter the definition of parthood for tuples to the following:

DEFINITION 1.7. (Generalised Parthood 2) For any two n-tuples $\vec{x}$, $\vec{y}$, $\vec{x} \preceq \vec{y}$ iff $\exists i : x_i \preceq y_i \wedge \forall j \neq i : x_j = y_j$.

This is a slightly odd way of generalising parthood to tuples, but it does allow for the situation in (20). Unfortunately, it is now no longer possible to read off the negative extension from the positive extension in the same way as previously. Assume a situation where some boys kissed some girls, but not all boys kissed anyone and not all girls were kissed. Concretely, say there is a plurality b_1 of boys who kissed all members of the plurality g_1 of girls, while no boy in b_2 kissed anyone and no girl in g_2 was kissed by anyone. Writing # for the third truth value, the relation *kissed* then looks like this:

kissed	g_1	g_2	$g_1 \oplus g_2$
b_1	1	0	#
b_2	0	0	0
$b_1 \oplus b_2$	#	0	?

Intuitively, (20) should be undefined in this scenario. However, the sentence could be simply false without violating homogeneity, since *kissed* isn't true of any of the four pairs that $\langle b_1 \oplus b_2, g_1 \oplus g_2 \rangle$ overlaps with (seen by checking whether there is a 1 in either the same row or the same column as the ?). Thus, given the formulation of homogeneity using definition 1.7, the negative extension of *kissed* is not as large as possible.

However, the following does work: use the definition of parthood in definition 1.5 and apply closure of the positive extension under pointwise fusion *after* checking for homogeneity. Note that in any case, the unary predicates obtained by keeping one argument fixed ($\lambda x.kissed(x, a)$ and $\lambda x.kissed(a, x)$, for any a) are always homogeneous.[13]

This issue disappears altogether if closure under pointwise fusion isn't assumed in the first place. In section 3.4, I will suggest an alternativ view on some of the examples that have been adduced to motivate the assumption that all lexical relations are closed under pointwise fusion. However, even if these suggestions should turn out to be on the right track and not all lexical relations are closed under pointwise fusion, there is still one that most definitely is: identity. If a is identical to itself (which it is) and b is also identical to itself, then $a \oplus b$ is identical to $a \oplus b$. For this to be the case, it is not required that $a = b$. Identity, as a special logical relation, might of course be exempt from homogeneity; but I will argue in section 4.4.1 that it is not. The existence of a homogeneity-based extension gap for identity, then, makes it necessary to have a story about how closure under pointwise fusion interacts with homogeneity.

13 Cf. section 2.4.1 for formal details.

1.1.5 *Interim Summary*

I have argued that homogeneity, which is traditionally viewed as the phenomenon that a predicate is neither true nor false of a plurality if it is true of some of its parts and false of others, can be found not only in distributive predicates, but also in many collective predicates,[14] and should be conceived of as a joint constraint on the positive and negative extensions of predicates: no plurality that a predicate is true of may overlap with a plurality that it is false of. This leads to new types of cases of undefinedness with collective predicates which have hitherto gone unnoticed. Given a positive extension, the negative extension of a predicate in natural language is as large as it can be without violating homogeneity. Thus, the positive extension alone suffices to derive the negative extension and the extension gap.

Binary relations show homogeneity effects as well: they are only false of a pair of pluralities if they are false of all pairs of parts of these pluralities. These can be captured if the notion of parthood is generalised to sequences of pluralities and it is assumed that closure of the positive under pointwise fusion is applied only after checking for homogeneity. The remainder of this chapter will, however, be concerned with unary predicates.

1.2 HOMOGENEITY REMOVAL

Löbner (2000) notes that the addition of *all*, either in the DP or in adverbial position, causes homogeneity to disappear. (21) is simple false whenever it is not true; there is no extension gap.

(21) (All) the professors (all) smiled.

TRUE iff every professor smiled.
FALSE iff at least one professors failed to smile.
UNDEFINED never.

Indeed, any quantifier has this effect: none of the sentences in (22) have an extension gap. *All* only happens to be somewhat special in that the removal of homogeneity is often essentially its sole semantic contribution.[15]

(22) a. Two/most/some of the professors smiled.
b. The professors mostly/partly smiled.

1.2.1 *Homogeneity and the Object Position*

Note, however, that quantifiers remove homogeneity only, as it were, with respect to their own argument position. If they fill the subject position of a transitive verb with a definite plural object, then homogeneity with respect to the object position still leads to an extension gap. (23a), for example, is undefined if every student

14 On those to which it doesn't apply, see section 1.4 below.
15 It does, however, still have a pragmatic contribution. See chapter 3 on this.

likes half of their siblings, but hates the other half. Similarly, (23b) is undefined if all but two students dislike all their siblings, and those two students like only half of their siblings.

(23) a. All the students like their siblings.
 b. Two students like their siblings.

Extension gaps of complex sentences due to an embedded definite plural are investigated in further detail in section 1.5. The pattern that is found is captured by the following statement of the truth and falsity conditions for *all*. *All a* is true of a predicate P is P is true of all parts of A, and it is false if P is false of at least one part of a.

(24) All a are P.

TRUE iff $\forall a' \preceq a : P(a') = 1$.
FALSE iff $\exists a' \preceq a : P(a') = 0$.
UNDEFINED otherwise.

The truth and falsity conditions for *two* are as follows: *two* X is true if there are two entities of which both X and P is true; and it it false if P is false of all X except for at most one.

(25) Two X are P.

TRUE iff $\exists x \exists y : x \neq y \wedge X(x) \wedge X(y) \wedge P(x) \wedge P(y)$.
FALSE iff $\exists x : X(x) \wedge \forall y : (x \neq y \wedge X(y)) \rightarrow \neg P(x))$.
UNDEFINED otherwise.

1.2.2 *Collective Predicates and Upward Homogeneity*

The formulations above account only for distributive predicates. But *two* is also compatible with collective predicates and readings.

(26) Two boys met.

In order to account for this, the truth conditions in (25) have to be slightly altered: the sentence is true if there is an individual sum consisting of two atomic X (a *duality* of X) such that P is true of that sum. The falsity conditions can remain unchanged, since negative predicates are always distributive.[16]

(27) Two X are P.

TRUE iff $\exists x : |x| = 2 \wedge X(x) \wedge P(x)$.
FALSE iff $\exists x : X(x) \wedge \forall y : (x \neq y \wedge X(y)) \rightarrow \neg P(x))$.
UNDEFINED otherwise.

This formulation makes a prediction for collective predicates: not all homogeneity is removed by quantifiers, in particular, upward homogeneity is retained in that it can give rise to extension gaps. In order to test this, a predicate has to be

16 That is to say, $\lambda x.\neg P(x)$ is a distributive predicate even if P is collective.

used that clearly brings out upward homogeneity. *Meet* is not an ideal choice for this purpose, since upward homogeneity is much more clearly visible with a predicate like *carry the piano upstairs* or *perform Hamlet*.

(28) Two boys carried the piano upstairs.

Of course, (28) has a distributive reading, too, but that one is somewhat absurd as it is a rare boy that can carry a piano upstairs on his own. What is of interest here is the collective reading. Consider a situation with three boys b_1, b_2 and b_3, and a girl g. The piano was carried upstairs by b_1, b_2, and g together. In this situation, by upward homogeneity, the predicate *carried the piano upstairs* is undefined of b_1, b_2 and their sum.[17] Consequently, there is no duality of boys that the predicate is true of, and the truth conditions of (28) are not fulfilled. But its falsity conditions aren't fulfilled, either: if there are three boys in total, then (28) is only false if the predicate is false of at least two of them; but it's only false of b_3. Thus, (28) is predicted to be undefined in a situation where two boys together with a girl carried the piano upstairs, which strikes me as correct.[18]

There is some disagreement about whether *all* and proportional quantifiers like *most* are compatible with such predicates as *carry the piano upstairs* and *perform Hamlet*, with whom upward homogeneity is clearly visible. According to Dowty's (1987), Brisson's (1998), and my own judgments, they are, but Dowty himself notes that other speakers disagree, and some other writers' intuitions align with theirs (Winter 2001, Champollion 2010).[19] There is, however, no doubt that *all* is compatible with such predicates once *together* is added. It may be that *together* influences the homogeneity properties of the predicate in some way — the situation is quite unclear to me — , but crucially, it is safe to say that it does not remove upward homogeneity. In a situation where all the boys together with the girls performed *Hamlet*, (29) is equally undefined with and without *together*.

(29) The boys performed *Hamlet* (together).

The natural hypothesis for truth and falsity conditions of *all* is then as in (30), where P is any predicate compatible with *all*, whether it may contain *together* or not.

(30) All a are P.

TRUE iff $P(a) = 1$.
FALSE iff $\exists a' \preceq a : P(a') = 0$.
UNDEFINED otherwise.

17 *Carry the piano* is possibly one of the collective predicates that can relatively easily be shifted to a participatory reading. This possibility must be disregarded here. Unfortunately, it is somewhat odd to combine *perform Hamlet*, which does not so easily lend itself to a participatory reading, with a numeral quantifier.

18 It should be mentioned that to obtain this judgment, the context should be imagined to be the question "What happened?". For some reason, there may be a stronger inclination to judge the sentence plainly false when the question is "Who carried the piano upstairs?". It is not clear to me why this is so.

19 The precise selectional restrictions of *all* are a complicated matter the investigation of which is beyond the scope of this work. Cf. also Moltmann 1997 and Champollion 2010.

Again, upward homogeneity is retained: (31) is predicted to be undefined in the situation described, since the predicate is undefined of all boys and pluralities of boys.

(31) *All of the boys together with some girls are performing* Hamlet*.*
All the boys are performing *Hamlet* (together).

Downward homogeneity and sideways homogeneity are removed. In particular, (32) should simply be false in the scenario given, which was used above as an illustration of sideways homogeneity. This seems plausible enough.

(32) *Some of the boys together with some of the girls are engaged in the performance.*
All the boys are performing *Hamlet* (together).

Note, however, that we are faced here with the curious case of a false sentence whose syntactic negation we might be reluctant to call straightforwardly true in the same scenario. The reason for this, I suggest, is that a negated *all*-sentence triggers the (non-negated) corresponding *some*-sentence as an implicature. In the given case, this implicature is not true, and in fact, it is not false, either, but undefined. A true sentence with an undefined implicature might plausibly lead to unstable judgments.

(33) *Some of the boys together with some of the girls are engaged in the performance.*
Not all the boys are performing *Hamlet* (together).
$\rightsquigarrow$ Some of the boys are performing *Hamlet* (together).

There is an interesting special case of sideways homogeneity. Assume a scenario where there are two performances of *Hamlet* going on simultaneously, and each of the boy participates in one of them (but not all of them in the same one). In such a situation, (34a) is undefined, and the generalisation in (30) predicts that (34b) is, too, which doesn't strike me as unreasonable.

(34) *Every boy is participating in one of multiple performances of* Hamlet*, and not all of them in the same one.*
a. The boys are performing *Hamlet*.
b. All the boys are performing *Hamlet*.

Note that in this case, the addition of *together* seems to make the sentence false. I have to leave open the question of what exactly the meaning of *together* is and how it interacts with homogeneity.[20]

1.2.3 *Interim Summary*

The intuitively formulated generalisation to be drawn from the observations in this section is that quantifiers remove downward homogeneity (and most instances of sideways homogeneity) with respect to their own argument position,

20 See Moltmann 2004 for a discussion of *together* without concern for homogeneity.

but not upward homogeneity, and not with respect to other argument positions in their scope (see section 1.1.3 on these terms). In chapter 2, I will discuss why this situation poses something of a challenge for a principled compositional treatment that makes general predictions for the truth and falsity conditions of arbitrary quantifiers and propose a solution.

1.3 VARIETIES OF HOMOGENEITY

So far, we have only seen homogeneity that is based on the relation of individual parthood — the relation that holds between a plurality and its individual parts. In truth, however, it is a much more general and pervasive phenomenon: it can be seen involving all manner of other notions of parthood, and it is not restricted to individuals, but can also be found with respect to objects in other domains. This section serves to give an overview of the breadth of the phenomenon, but does not aim at in-depth analysis. Some particular instantiations of homogeneity will be taken up and subjected to more detailed investigation in chapter 7.

1.3.1 *Material, Spatial, and Other Parts*

It was already noticed by Löbner (2000) that predicates can be homogeneous with respect to various different notions of parthood. (35), for example, shows homogeneity with respect to sections of the wall. The sentence (35a) and its negation (35b) are both neither true nor false when only part of the wall is painted red (and the rest is as yet untreated).

(35) a. The wall is painted red.
b. The wall isn't painted red.

Similarly, homogeneity in (36) must be evaluated with respect to spatially connected parts of the forest. In some sense, any arbitrary collection of trees is a part of the forest; but not all such collections count for the purpose of homogeneity. For one can always choose some set of trees in the forest that stand far apart and therefore don't constitute something that is dense. What is considered for the purpose of homogeneity is only connected subregions of the forest. When the forest isn't dense throughout, but has regions where it isn't, then a homogeneity violation ensures.[21]

(36) The forest is/isn't dense.

Entirely abstract entities, too, have parts with respect to which a predicate that applies to such abstract entities can be homogeneous. When a book contains both brilliant and stupid chapters, (37) incurs a homogeneity violation and fails to be true or false.

(37) The book is intelligently written.

21 Note that this is to be distinguished from a situation where the density is constant across the whole forest, but the density value is such that it's a borderline case of *dense*.

These other dimensions of homogeneity have their own homogeneity removers that work analogously to adnominal and adverbial *all*. Adnominal *whole* is applicable in all the cases.[22]

(38) a. The whole wall is(n't) painted.
b. The whole forest is(n't) dense.
c. The whole book is(n't) intelligently written.

In the spatially flavoured cases, *everywhere* functions as an adverbial homogeneity remover.

(39) a. The wall is(n't) painted everywhere.
b. The forest is(n't) dense everywhere.

For the very abstract case of the book, *throughout* may be regarded as an adverbial homogeneity remover in English, but for some reason, it seems to sound slightly odd when combined with negation.

(40) a. The book is intelligently written throughout
b. ?The book isn't intelligently written throughout.

In German, however, there is the adverbial *durchgehend* that is usable both with and without negation just as naturally as *everywhere*.

(41) a. Das Buch ist (nicht) durchgehend intelligent geschrieben.
the book is (not) throughout intelligently written

These, of course, are only the universals. There are, in fact, adnominal and adverbial quantifiers of various strengths, such as *part of*, *most of*, *partly*, *in large part*, and many others, which are all associated with one or more kinds of parthood relations. Here, again, we find cross-linguistic variation in which items are available and can be used for which dimension.

Note that homogeneous predications can can, in a sense, be nested:[23] a predicate that is homogeneous with respect to abstract parts can, for example, be distributively predicated of a plurality of objects, as in (42). Then there are, as it were, two layers homogeneity: the outer layer of homogeneity with respect to the individual parts of the plurality of all books, and the inner layer with respect to the abstract parts of the individual books. One would then predict that (41) is true if all the books are intelligently written throughout, and false if no book contains any intelligent sections.

(42) The books are intelligently written.

These two layers can be removed separately: in (43a), only the homogeneity of the distributive plural predication is removed, in (43b), only the homogeneity of

22 How it comes to be an adjective that forms a constituent with the noun below the definite article, instead of attaching to the whole definite DP like *all*, is, of course, mysterious in the same way as *die meisten* 'most' is in German. They are generally analysed as complex determiners. See also Moltmann 1997 on *whole*.

23 Cf. also Löbner 2000.

the lexical predicate with respect to the parts of individual books is, and in (43c), there is no homogeneity whatsoever.

(43) a. All the books are intelligently written.
b. The books are intelligently written throughout.
c. All the books are intelligently written throughout.

Note that that an adnominal homogeneity remover cannot be used to remove the inner layer of homogeneity if an outer layer is also present.

(44) a. ??The whole books are intelligently written.
b. #All the whole books are intelligently written.

It seems plausible that all of this can be modelled in a multi-sorted ontology in which objects have aspects of different ontological categories (see e. g. Asher & Pustejovsky 2000, Asher 2011, Retoré 2014) and different parthood relations on these sorts are taken into account by extending the approach of Link 1983. The detailed formal development of such a picture, taking into account the results from chapter 2, will have to remain as a task for future research.

1.3.2 *Homogeneity in the World Domain*

It has variously been observed that to deny a conditional or call it false or impossible seems to be the same thing as asserting a version of the sentence where the consequent is negated. This is sometimes referred to as the *conditional excluded middle* (Stalnaker 1981, von Fintel 1999).

(45) A: If Nina comes, Adam will be happy.
B: No, I don't think so.
$\rightsquigarrow$ If Nina comes, Adam won't be happy.

If one merely wishes to point out that even if Nina comes, it is still possible that Adam will not be happy, i. e. that the conditional isn't *true*, something weaker is needed.

(46) A: If Nina comes, Adam will be happy.
B: Well, not necessarily.

This suggests that conditionals are homogeneous in something like the following way, and that *necessarily* is the applicable homogeneity remover.[24]

(47) If p, q.

TRUE iff in all accessible p-worlds, q is true.
FALSE iff in all accessible p-worlds, q is false.
UNDEFINED otherwise.

24 It seems that *necessarily* requires the presence of negation. Unnegated uses seem to be unique to philosophers' discourse and are somewhat unidiomatic in ordinary English.

This will be argued at greater length in chapter 7. The observations fit well with suggestions that have been made of analysing conditional antecedents as referential and denoting pluralities of antecedent worlds (Schlenker 2004).

In chapter 6, I will furthermore suggest that the phenomenon of neg-raising can perhaps also be viewed in terms of homogeneous predication over pluralities of worlds.

1.3.3 *Homogeneity in the Kind Domain*

Bare plural generics show a homogeneity effect just like definite plurals (Löbner 2000, Cohen 2004, Magri 2012): there is a gap between (48a) and (48b), which contains the situations where the fact of the matter is that some (kinds of) dogs are intelligent and others aren't. Both sentences seem to be neither true nor false once it is acknowledge that there are intelligent and unintelligent kinds of dogs.

(48) a. Dogs are intelligent.
  b. Dogs aren't intelligent.

The addition of *all* again has the effect of removing homogeneity: (49) is plainly false as soon as there are stupid dogs.

(49) All dogs are intelligent.

For a more detailed discussion of homogeneity in generics and its relationship with their exception tolerance, the reader is referred to section 7.2.

1.3.4 *Homogeneity in the Event Domain*

Imagine that Dennis boarded a ship in Southampton on Monday and arrived in New York on Sunday. What, then, is the status of (50)?

(50) Dennis travelled to New York on Thursday.

The sentence is certainly not true; only its progressive cousin (51) is.

(51) Dennis was travelling to New York on Thursday.

But one is also hesitant to call false. The most appropriate answer would seem to be neither *yes* nor *no*, but *well* followed by a correction, which is the pattern found with homogeneity violations.

(52) Context: *Dennis's journey to New York took from Monday to Sunday.*
  A: Dennis travelled to New York on Thursday.
  B: #Yes / #No / ✓Well, he was en route.

Assume that the logical form of (52) is something along the lines of (53), as it might be in a simplistic version of event semantics: it says there is an event which is a travelling of Dennis to New York and which takes place on Thursday.

(53) $\exists e : travel(e, Dennis, New\ York) \wedge on(e, Thursday)$

There are different possible diagnoses now, which may all be correct at the same time. It is clear that the sentence is not true, so we're interested in the reason for its undefinedness. There are two ways of explaining this in terms of homogeneity in the event domain. One could say that there is an event which happened on Thursday of which $\lambda e.travel(e, Dennis, New\ York)$ is undefined due to upward homogeneity, rendering (53) undefined. Another way to look at it is to say that there is a travelling to New York by Dennis of which $on(e, Thursday)$ is undefined by downward homogeneity, because part of that event happened on Monday and part of it happened on other days.[25]

There doesn't seem to be a homogeneity remover like *all* in the event or interval domain, but there are plenty of predicates of events which are clearly non-homogeneous, such as *in two days* or *from Monday to Wednesday*. Both (54a) and (54b) are plainly false, even though there are subevents of Dennis's journey of which these predicates are true.

(54) a. Dennis travelled to New York in two days.
b. Dennis travelled to New York from Monday to Wednesday.

A more detailed investigation of verbal semantics with regard for homogeneity will eventually have to show whether it is more appropriate to think of this as homogeneity in the event domain propert, or as homogeneity with respect to temporal interval inclusion, or whether the two approaches aren't wholly equivalent anyway.

1.3.5 *Homogeneity in the Temporal Domain*

Temporal *when*-clauses are very similar to *if*-clauses, so one might expect them to show the same behaviour with respect to homogeneity. Indeed, they are also very similar to definite descriptions. The meaning of (55) intuitively involve a number-neutral definite description of something like time intervals.

(55) John was happy when he visited his sister.
'John was happy at the time(s) when he visited his sister.'

If there is a unique salient visit of John's to his sister, then this behaves like a singular definite description and there is no homogeneity to be expected. But assume that we are talking about a longer period of time during which John visited his sister several times. Then (55) is true if he was happy every time he visited her, but false only if he wasn't happy during any of the visits.

(56) Context: *During some, but not all visits to his sister, John was happy..*
A: John was happy when he visited his sister.

25 In principle, things could also be phrased in terms of homogeneity with respect to interval inclusion in the time domain, since the reason why $on(e, Thursday)$ is undefined might be upward homogeneity coupled with the fact that $\lambda t.on(e, t)$ is true of a superinterval of Thursday (namely the interval that lasts from Monday to Sunday).

B: #Yes / #No / Well, sometimes.

A non-homogeneous version of the same thing can be formed by adding *always* as an adverbial quantifier or using *whenever* instead of *when*. In those cases, the existence of a single unhappy visit enables a denial with *no*.

(57) A: John was always happy when he visited his sister.
A′: John was happy whenever he visited his sister.
B: No, there was that one time where she made him absolutely furious.

1.3.6 *Homogeneity with Embedded Questions*

A *wh*-question embedded under *know*, as in (58a), normally leads to a sentence that, in order to be true, at least requires the subject to know that all true answers are true, and most likely something even stronger.[26]

(58) Agatha knows who was at the party.

In a situation where Agatha has correct beliefs except that she wrongly believes Nina and Adam, who were, in fact, present, to have been absent, (58) is not true; but neither would we say that its negation (59) is. Both sentences seem to fail to have a definite binary truth value.

(59) Agatha doesn't know who was at the party.

The picture we find seems to be roughly like this

(60) Agatha knows who was at the party.

TRUE iff Agatha is fully informed about who was (and wasn't?) present.
FALSE iff Agatha has no idea who was (and wasn't?) present.
UNDEFINED otherwise.

English has adverbial quantifiers like *mostly* that can target embedded questions, but perhaps no direct equivalent to *all* that just closes the extension gap. German, however, does have an element that, when added inside the question, has this effect.

(61) Agathe weiß, wer aller auf der Feier war.
Agatha knows who ALL *at the party was*

TRUE iff Agatha is fully informed about the guests.
FALSE iff there was a guest Agatha doesn't know about.
UNDEFINED otherwise.

26 What, precisely, the additional requirement is is the subject of a debate that is of no immediate concern at this point. Cf. e. g. George 2011, Klinedinst & Rothschild 2011, Égré & Spector 2014, and references therein.

This is strongly suggestive of homogeneity as one sees it with definite plurals.[27] What I have presented here is, of course, only the barest of outlines. For more details and further substantiation, see chapter 7.

1.3.7 *Homogeneity and Conjunction*

Conjunctions of individual-denoting terms are traditionally analysed via shifting the individuals to the quantifier type and then applying a generalisation of boolean conjunction to it, so that the end result is just the conjunction of two propositions which are formed by combining the predicate with either of the disjuncts (cf. e. g. Winter 2001).

(62) $[\![\text{John and Mary}]\!] = \lambda P.P(John) \wedge P(Mary)$

This cannot, however, explain that such conjunctions can also receive collective readings: (63) has as its most prominent reading one on which it conveys that John and Mary together carried the piano upstairs.

(63) John and Mary carried the piano upstairs.

This leads to the idea that at least one potential meaning of conjunction must be the formation of the mereological sum of the conjuncts, and it has been argue that this is, in fact, the only meaning for conjunction, which can also be generalised across types by the right algebraic means (Schmitt 2012b). If conjunctions of individual terms denote individual sums, then they should give rise to homogeneity effects: (64) should be undefined if, as is in fact the case, the character of Adam appears only in one of the two books.

(64) Adam appears in *Decline and Fall* and *Vile Bodies*.

The existence of such a homogeneous conjunction at least for individuals has been argued for by Szabolcsi & Haddican (2004) and proposed for all domains by Schmitt (2012b). A conclusive empirical verdict on the latter claim in particular seems to be still wanting, and it is also not quite clear how prevalent homogeneous conjunction is in English even for individuals.

What is well-known is that there is at least one condition under which conjunction is unambiguously non-homogeneous: when it bears contrastive stress. (65) is clearly true in a situation where Sandy read one of the books, but not both.

(65) Sandy didn't read *Decline and Fall* AND *Vile Bodies*.

The exact mechanism for this process is not clear, but it seems natural to assume that it is related to the fact that presuppositions can be locally accommodated with stess on the trigger and that scalar items can be locally exhaustified when

27 This parallel has been noticed by a number of people, including Benjamin George and Benjamin Spector (p. c.), but has not, to my knowledge, been discussed in print.

contrastively stressed.[28] Gradable adjectives also show a gap-like phenomenon that disappears under stress in the same way.

(66) a. John isn't tall. $\rightsquigarrow$ John is rather short.
b. It's not likely that John will win. $\rightsquigarrow$ It's unlikely that John will win.

(67) a. John isn't TALL, but he's not SHORT, either.
b. It's not LIKELY that John will win, but it's quite possible. I'd say the chances are about fifty-fifty.

All of these may be instances of a phenomenon that is usually discussed under the label of *metalinguistic negation* (Horn 1985, Geurts 1998), the intuition behind it being that what is being negated is not the not literally the semantic content of the stressed expression, but rather the appropriateness of its use.

Such stress-based homogeneity removal under negation is not found with definite plurals, which can intuitively be made sense of based on the following fact. The definite article is very difficult to stress, and if it is stressed, it can only be contrasted with the indefinite article. Doing so, as in (68), conveys that there is no specific salient plurality of books for the definite to refer to.

(68) Adam didn't read THE books, he read SOME books.

But in order to remove just remove homogeneity, while keeping the reference, what we would need is the contrast with *some of the*. This alternative isn't available because it properly contains *the*. One would, effectively, want to contrast $\varnothing$ with *some of*, but of course $\varnothing$ cannot be stressed and so this is impossible. This, I submit, is why no analogue to stressed *and* can be found for definite plurals and other homogeneous constructions.

1.4 NON-HOMOGENEOUS PREDICATES

Some predicates in natural language are not homogeneous in any way. Conforming to homogeneity would, in fact, completely destroy the intuitive meaning of these predicates. Consider, for example, the predicate *numerous*. If it were homogenous, that would mean that if (69) is true, *numerous* cannot be false of any subgroup of the children. More generally, the predicate couldn't actually ever be false of any plurality. For surely the sum of all individuals in the model is a numerous plurality, and every other plurality is a part of it, of which, by homogeneity, the predicate then couldn't be false. This is obviously absurd.

(69) The children are numerous.

28 Apparently local presupposition accommodation is exemplified in (ia), while local exhaustification is observed in (ib).

(i) a. Adam didn't STOP smoking — he never smoked in the first place!
b. Adam didn't eat SOME of the apples — he ate ALL of them!

The mirror image of these considerations applies to *few in number*. The collective readings of *heavy* and *light* are also non-homogeneous, and must be, for the same reason. Talking about a heap of grains on a scale, one may say (70), but surely this does not mean that *heavy* isn't plainly false of any individual grain.

(70) The grains are heavy.

German has a construction that corresponds to a version of *numerous* with overt measure phrases. The predicate *zu n-t sein* translates roughly as *be a group of n*.

(71) Die Kinder sind zu zweit.
the children are ZU 2-T
'The children are a group of two.'

Obviously, these predicates are also non-homogeneous. It has been suggested that all these predicates do not actually take pluralities as their arguments, but a different kind of object: a group, which, as far as the relation of individual parthood is concerned, is an atom (Champollion 2010). It is, however, not clear that this is in any way helpful for explaining why they are not homogeneous, since doubtlessly there is some sort of parthood relation that holds between a group individual and its subgroups, even if it is not the same one that holds between pluralities.

1.4.1 *Non-Homogeneity and Measure Functions*

There is, however, something potentially more interesting to be said about these predicates. It is remarkable that all of them seem to involve some sort of measure function; more precisely, a measure function that is monotonic with respect to individual parthood. If a is a proper part of b, for example, then the number of atoms in a or the weight of a is strictly smaller than the number of atoms in or the weight of b, respectively. Numerous, for example, seems to mean something like (72), where $|\cdot|$ is the measure function that returns the number of atoms in a plurality and s is some contextual standard. If this predicate is to have both a positive and a negative extension that is non-empty, it cannot be homogeneous.

(72) $[\![\text{numerous}]\!] = \lambda x.|x| \geq s$

It is worth noting that the homogeneity-removing quantifiers discussed in section 1.2 are usually also thought of in terms of measure functions. No precisely analogous reasoning is possible, but there is at least something in that direction. Consider, for example, the following meaning for *mostly*.

(73) $[\![\text{mostly}]\!] = \lambda P.\lambda x.\exists y \prec x : \frac{|y|}{|x|} > 0.5 \wedge P(y)$

Now if this predicate were homogeneous, then (74) would only be false if none of the invitees came, which intuitively seems bizarre.

(74) The people invited mostly came.

Interestingly, there is a case to be made that existential bare plurals are the only quantifiers that do not remove homogeneity, and conspicuously, they don't involve a numeral or other visible indication of measurement being applied. In chapter 5, I will explore the idea that perhaps the multiplicity inference with bare plurals, which disappears under negation, is another instance of homogeneity.

(75) Mary saw zebras.

TRUE if Mary saw more than one zebra.
FALSE if Mary saw no zebra at all.
UNDEFINED if Mary saw exactly one zebra.

This suggests the very intuitive generalisation that whenever homogeneity disappears in some way, it is because such a monotonic measure function is involved. An first attempt at an implementation of this idea is made in chapter 2.

1.5 HOMOGENEITY IN COMPLEX SENTENCES

If the application of homogeneous predicates to pluralities gives rise to an extension gap, then it is natural to ask how this plays out in complex sentences with scope-taking elements. What is the extension gap of a complex sentence where a definite plural is embedded in the scope of a quantifier? This is analogous to the classical problem of presupposition projection, and I will therefore sometimes refer to it as the problem of *homogeneity projection*.

The experiments by Križ & Chemla (2015) have supplied us with a much more detailed picture for homogeneity than is available for presuppositions. They investigated the extension gaps of sentences of the form in (76) for the quantifiers *all/every*, *some*, and *exactly two* by presenting subjects with schematic depictions of a situation and asking them to judge a sentence as *completely true*, *completely false*, or *neither*.

(76) [Quantifier] of the boys found their presents.

The same sentences with *all of their presents* instead of the simple definite descriptions functioned as controls in order to ascertain that *neither* judgments were really a reflection of the extension gap rather than some intuitive notion of closeness to truth (since the *all*-sentences are true under just the same conditions).

A guiding intuition in choosing configurations to investigate as potential gap cases is this: a definite truth value is obtained whenever it doesn't matter which way the gap cases are resolved.[29] If resolving the gap cases in different ways leads to different truth values, then the whole sentence is undefined. To see how this plays out, consider the evaluation of the sentence (77) in the two scenarios in (78).

(77) Every boy found his presents.

29 This is, of course, strongly related to the intuition behind Strong Kleene logic. The heuristic tracks the generalisation of Strong Kleene logic to quantification over atoms. Cf. also section 2.1.1.

(78) Scenario 1: *One boy found all of his presents, the second found half, and the third found none.*

Scenario 2: *One boy found all of his presents and the two others each found half.*

(79) provides schematic representations of the extension of *found his presents* in these scenarios, with # as the third truth value. What is meant by "resolving gap cases" is simply replacing # in these functions by either 0 or 1.

(79) Scenario 1: $[\![\text{found his presents}]\!] = \begin{bmatrix} s_1 \mapsto 1 \\ s_2 \mapsto \# \\ s_3 \mapsto 0 \end{bmatrix}$

Scenario 2: $[\![\text{found his presents}]\!] = \begin{bmatrix} s_1 \mapsto 1 \\ s_2 \mapsto \# \\ s_3 \mapsto \# \end{bmatrix}$

In the first scenario, there is one boy in the extension gap of the restrictor predicate. There are two ways of resolving the gap case, yielding the functions in (80a) and (80b), respectively.

(80) a. $\begin{bmatrix} s_1 \mapsto 1 \\ s_2 \mapsto 1 \\ s_3 \mapsto 0 \end{bmatrix}$

b. $\begin{bmatrix} s_1 \mapsto 1 \\ s_2 \mapsto 0 \\ s_3 \mapsto 0 \end{bmatrix}$

But for both of them, the quantifier *every boy* yields the same truth value: it is false, because there is always at least one boy of whom the scope predicate *found his presents* is false. Hence the sentence (77) is simply false. In scenario 2, it does matter how we resolve the gap cases: *every boy* is false of the resolution in (81a), but true of the resolution in (81b). Consequently, the sentence is undefined.

(81) a. $\begin{bmatrix} s_1 \mapsto 1 \\ s_2 \mapsto 1 \\ s_3 \mapsto 1 \end{bmatrix}$

b. $\begin{bmatrix} s_1 \mapsto 1 \\ s_2 \mapsto 0 \\ s_3 \mapsto 0 \end{bmatrix}$

Note that while this heuristic using resolutions of undefined cases is convenient, and also successful, for distributive predicates, it will become apparent in the next chapter that it cannot, unfortunately, straightforwardly form the basis of a formal logic for homogeneity projection.

1.5.1 *Homogeneity under* all/every

The experimental results found for homogeneity projection from under *all* and *every* exactly match the guiding intuition.

(82) All of the boys found their presents.

TRUE iff all of the boys found all of their presents.
FALSE iff at least one boy found none of his presents.
UNDEFINED otherwise.

1.5.2 *Homogeneity under* exactly two

In the case of a non-monotonic quantifier, one can identify three interesting types of situations in which the heuristic predicts a lack of truth value.

(83) Scenario 1: *One boy found all of his presents, the second found half, and the third found none.*

Scenario 2: *One boy found all of his presents, and the other two found half of them.*

Scenario 3: *Two boys found all of their presents and the third found half of them.*

Schematically, these can be represented as follows.

(84) Scenario 1: $[\![\text{found his presents}]\!] = \begin{bmatrix} s_1 \mapsto 1 \\ s_2 \mapsto \# \\ s_3 \mapsto 0 \end{bmatrix}$

Scenario 2: $[\![\text{found his presents}]\!] = \begin{bmatrix} s_1 \mapsto 1 \\ s_2 \mapsto \# \\ s_3 \mapsto \# \end{bmatrix}$

Scenario 3: $[\![\text{found his presents}]\!] = \begin{bmatrix} s_1 \mapsto 1 \\ s_2 \mapsto 1 \\ s_3 \mapsto \# \end{bmatrix}$

In all three of these situations, it depends on how the gap cases are resolved whether (85) is true.

(85) Exactly two of the boys found their presents.

All of these situations were tested experimentally. Scenarios 1 and 3 have been confirmed as gap cases, but interestingly, Scenario 2 does not show up as such. This suggests that the heuristic needs to be modified: when gap cases are resolved, they all have to be resolved in the same direction. Thus, for scenario 2, one does not consider the resolution where one gap case is counted as true and the other as false.

1.5.3 *Homogeneity under* no

For (86), the heuristic predicts a gap whenever at least some boys found some of their presents, but none of them found all.

(86) No boy found his presents.

In the experimental results, the proportion of *neither* judgements is noticeably smaller, but it is still robustly present. The heuristic, of course, makes only qualitative predictions, as will the logic to be developed in chapter 2. The considerations to be presented in chapter 3 may help to make sense of quantitative variation.

1.5.4 *Homogeneity in Restrictors*

What has not yet been experimentally tested is what happens when a homogeneous predication is embedded in the restrictor of a quantifier, such as in (87).

(87) Everybody who solved the math problems passed the exam.[30]

The heuristic based on resolutions of gap cases makes the following predictions.

(88) TRUE iff everybody who solved any of the math problems passed.
FALSE iff somebody who solved all of the math problems didn't pass.
UNDEFINED otherwise.

This is certainly not what (87) means. But on the other hand, there are situations where an existential interpretation of a definite plural inside a restrictor is indeed observable. (89), for example, does not only speak about those who touched all of the status, but is normally understood as saying that anybody who touched any statue had to leave.

(89) Everybody who touched the statues was asked to leave.

It could be that homogeneity projection from restrictors is optional, and that for (89), the reading with projection is more prominent, whereas for (87), it is the one without. This would predict the right truth conditions for (89), but the falsity conditions that follow are overly strong: the sentence would only be false if someone who touched all the status was allowed to stay. According to my judgment, however, it is false as soon as anybody who touched any statue wasn't asked to leave.

Note that the same two kinds of readings can also be found in the antecedent of conditionals, which have often been assumed to be restrictors of a quantifier over worlds.[31]

(90) a. If you solve the problems, you will pass the exam.
'If you solve all of the problems, you will pass the exam.'
b. If you touch the statues, you will be asked to leave.
'If you touch any of the statues, you will be asked to leave.'

In chapter 3, I will propose an explanation for the seemingly existential readings of definite plurals in downward-entailing restrictors that is based on pragmatics

30 For a context, imagine that the exam contains questions from various fields, but being able to solve the math problems is a very good predictor of overall performance.

31 This view originates with Lewis 1975, Kratzer 1981, 1986, and Heim 1982. For recent discussion, see Rothschild forthcoming.

and does not assume a separate semantic reading of the sentences in question. If this is correct, then it seems that homogeneity never projects from restrictors.

A further argument for this comes from quantifiers with upward-entailing restrictors. The heuristic I have been employing makes the following prediction.

(91) Two students who solved the math problems passed.

TRUE iff there are two students who solved all the math problems and passed.
FALSE iff no more than one student who solved any of the math problems passed.
UNDEFINED otherwise.

These falsity conditions strike me as incorrect: in a situation where only one student solved all of the math problems and passed, but a second student solved some of the problems and also passed, (91) still seems just false. This also points towards non-projection of homogeneity from restrictors. Note that it is possible for definite plurals to receive effectively existential readings in upward-entailing restrictors as well. (92) does not entail by any means that the two students touched all of the statues.

(92) Two students who had touched the statues were asked to leave.

1.5.5 *Interim Conclusion*

Sentences with a definite plural in the scope of a quantifier show extension gaps according to a particular pattern, which has been experimentally demonstrated by Križ & Chemla (2015). In analogy to presupposition projection, one may call this phenomenon *homogeneity projection*. The intuitive heuristic that traces the pattern that was found experimentally is that the truth value of the sentence has to be the same now matter how the undefined cases have to be resolved. While there are no experimental data for definite plurals in the restrictor of quantifiers, it can be shown that homogeneity does not project from restrictors in the same way. The trivalent logic to be presented in chapter 2 captures these patterns.

1.6 THE IMPLICATURE THEORY OF HOMOGENEITY

The first theoretical work to deal with the issue of homogeneity projection with any detail was Magri 2014, which proposes a view of homogeneity that is at odds with the trivalent perspective pursued here. On this theory, the semantic truth conditions of (93a) and (93b) are, in fact, complementary, and the appearance of a gap arises because one of the sentences is strengthened by a scalar implicature.

(93) a. Adam wrote the books.
b. Adam didn't write the books.

Magri implements scalar implicatures by means of an exhaustivity operator.[32] What this operator does it to negate all alternatives that are not entailed by the original sentence. Note that this includes not only stronger alternatives, but also the logically independent ones.

DEFINITION 1.8. (Exhaustivity Operator)

$$[\![\textsc{exh}(\phi)]\!] = [\![\phi]\!] \cap \bigcap\{[\![\neg\psi]\!] \mid \psi \in \text{Alt}(\phi) \wedge \phi \nrightarrow \psi\}$$

In order to derive homogeneity effects, Magri makes the following further assumptions. First, the literal meaning of definite plurals is that of an existential quantifier. Second, there are the two scales of alternatives in (94), and, importantly, alternativehood is not transitive across scales. Thus, *the* and *all of the* are not alternatives of each other, even though both are alternatives of *some of the*.

(94) a. *some* $\sim$ *all*
     b. *the* $\sim$ *some of the*

The last important ingredient is that the exhaustivity operator is, in fact, applied not once, but twice at the top of every sentence.[33] Here is, then, how this plays out. By definition of EXH, (95a) means the same thing as the conjunction of (95b) with the negations of all the non-weaker alternatives of (95b).[34]

(95) a. EXH(EXH(Adam wrote the books)
     b. EXH(Adam wrote the books)

To know what the meaning of (95b) is, we need to consider the two alternatives of (96), which are (96a) and (96b).

(96) Adam wrote the books.
     a. Adam wrote the books.
     b. Adam wrote some of the books.

By assumption, (96a) has an existential meaning, and so these two alternatives are, in fact, synonymous. The single exhaustivity operator applied to a sentence with a definite plural is therefore vacuous.

(97) EXH(Adam wrote the books)
     = Adam wrote the books
     = Adam wrote some of the books

Now we know the meaning of (95b). To obtain the meaning of (95a), we now need to consider the alternatives of (95b), which can be seen in (98).

32 This is the standard practice in what is known as the grammaticalist tradition about scalar implicatures. Magri's theory does not, however, make use of local implicatures and is therefore not *fundamentally* at odds with a neo-Gricean view, as long as the global implicature calculation process yields the same results as his (multiple, as will soon be seen) application of the exhaustivity operator.

33 This is a common practice in the grammaticalist tradition on implicatures, cf. e. g. Spector 2006 and Fox 2007.

34 In the discussion to follow, I will not distinguish between sentences and their denotations.

(98) EXH(Adam wrote the books)
  a. EXH(Adam wrote the books)
  b. EXH(Adam wrote some of the books)

The meanings of those alternatives are clear:

(99) a. EXH(Adam wrote the books)
   = Adam wrote some of the books
  b. EXH(Adam wrote some of the books)
   = Adam wrote some but not all of the books

(99b) logically entails (99a), so it is negated when computing the meaning of (95a).

(100) EXH(EXH(Adam wrote the books)
  = EXH(Adam wrote the books) and
   not(EXH(Adam wrote some but not all of the books))
  = Adam wrote some of the books and
   Adam didn't write some but not all of the books
  = Adam wrote all of the books

The general pattern is given in (101). For detailed derivations of the implicatures for definite plurals in various embedded contexts, the reader is referred to the original paper.

(101) $\text{EXH}(\text{EXH}(\phi(\text{the}))) = \begin{cases} \phi(\text{some}) & \text{if } \phi(\cdot) \text{ is downward-entailing,} \\ \phi(\text{some}) \wedge \phi(\text{all}) & \text{otherwise.} \end{cases}$

The implicature theory of homogeneity rests on a set of very specific and by no means uncontentious assumptions, first and foremost perhaps the assumption that alternatives are allowed to be contained in one another and that alternative-hood is not transitive.[35] One may or may not be sympathetic to these ideas, but of course the proper course of action is to identify and test predictions that follow from them.

1.6.1 *The Status of the Gap*

So far I have always spoken of homogeneity in terms of an extension gap, which is something that an implicature-based theory does not actually supply. Magri does not discuss this issue, but there is an intuitive way in which a sentence can have something of both truth and falsity about it: it could be that the literal meaning of the sentence is true, but its scalar implicature is false. And indeed Križ & Chemla 2015 found that at least in a certain percentage of cases, speakers judge a sentence with a true literal meaning but a false implicature to be *neither*

35 The assumption that alternatives can contain each other seems to be somewhat at odds with theories of alternatives such as Katzir's (2007), according to which alternatives must be at most equally, but no more, complex.

completely true nor completely false in the same way as a sentence that incurs a homogeneity violation.

In a situation where none of the triangles is, in fact, green, the literal meaning of (102) is true, but its (indirect) implicature that some of the triangles are green is false. Križ & Chemla found that in such a situation, speakers judged the sentence to be *completely false* approximately 25% of the time and to be *completely true* 50% of the time. They chose the reply *neither* in the remaining quarter of cases.

(102) Not all the triangles are green.

Thus, is could be that the *neither*-judgments for (103) in a situation where some, but not all of the triangles were green were due to the fact that the literal meaning (103a) was true while the implicatures (103b) was false.

(103) The triangles are green.
  a. Some of the triangles are green.
  b. All of the triangles are green.

It should be noted that the proportion of gap-judgments was much larger here: subjects chose *neither* in about 50& of the cases, and *completely true* (the literal meaning) only around 10% of the time. However, the percentages varied to some extent between different embedding contexts, and a defender of the implicature theory would likely argue that not all implicatures have the same probability of being drawn. Some are more plausible than others. The most direct possible test for this would be to compare (104a) and (104b).

(104) a. The triangles are green.
  b. Some of the triangles are green.

Since the implicature of (105b) from *some* to *not all* is a subcomputation in the computation of the implicature of (105a) from *the* to *all of the*, the former should not occur more infrequently than the latter. This experiment, however, has not been performed, and I will continue to treat the data in a merely qualitative manner as identifying the presence or absence of a gap.

1.6.2 *Homogeneity Projection*

Magri's theory was, to my knowledge, the first to make substantive predictions about the behaviour of homogeneity in complex sentences, and is quite successful in doing so. A detailed discussion and evaluation of this aspect of the theory can be found in Križ & Chemla 2015, but the main points will be reviewed here.

Assume that a gap (in the relevant sense) arises whenever a sentence and its implicature have opposite truth values. For a context $\phi(\cdot)$ that is not downward-entailing, these two meaning components are as follows.

(105) Literal Meaning: $\phi(\mathit{some})$
  Implicature: $\neg(\phi(\mathit{some}) \wedge \neg\phi(\mathit{all}))$, equivalently $\neg\phi(\mathit{some}) \vee \phi(\mathit{all})$

1.6.2.1 *Upward-Entailing Contexts*

The prediction for *all* is quite straightforward: (106) has a gap where every boy found some of his presents, but not every boy found all of his presents.

(106) All the boys found their presents.

This is exactly what Križ & Chemla found.

1.6.2.2 *Non-Monotonic Contexts*

Due to the fact that the exhaustivity operator excludes not only stronger alternatives, but also non-weaker alternatives, definite plurals in a non-monotonic context are also predicted to give rise to implicatures and hence to gap judgments. (107) has the literal meaning in (107a) and the implicature in (107b).

(107) Exactly two of the boys found their presents.
  a. Exactly two of the boys found some of their presents.
  b. Exactly two of the boys found all of their presents.

In section 1.5.2 above, I discussed three different kinds of situations as potential candidates for gap situations.

(108) Scenario 1: *One boy found all of his presents, the second found half, and the third found none.*

In this scenario, the literal meaning in (107a) is true, but the implicature (107b) is false. A gap is predicted, and was found.

(109) Scenario 2: *One boy found all of his presents, and the other two found half of them.*

Here, both the literal meaning and the implicature are false, and so no gap is predicted. This is, again, in line with the experimental findings.

(110) Scenario 3: *Two boys found all of their presents and the third found half of them.*

In this situation, the literal meaning of the sentence is false, but its implicature is true. Since it was stipulated that it is a *difference* in truth value between the literal meaning and the implicature that gives rise to a gap judgment, the fact that a gap was found in these situations can be accounted for.

1.6.2.3 *Downward-Entailing Contexts*

The implicature theory predicts that a definite plural in a downward-entailing context should never give rise to an extension gap. It is just interpreted as an existential and there is no implicature that could differ in truth value from the literal meaning.

Križ & Chemla's experimental stimuli with sentential negation unfortunately had the definite plural in subject position, which makes the results not decisive. If the definite plural is interpreted with surface scope, then it is, in fact, in an upward-entailing context and the gap judgments can be explained. A sentence with the definite plural in object position would be more informative, but one would not intuitively expect it to show different results. This would pose something of a problem for the implicature theory. One potential way out might be to claim that for some reason, an inverse-scope reading, which again has the definite plural in an upward-entailing context, is extremely available. I do not find this particularly convincing, but it is important to note that this makes a prediction: there should be no gap at all if the definite plural is embedded under a negative quantifier and contains a pronoun bound by that quantifier.

(111) No boy found his presents.

Here, no scope inversion can take place, since scoping the definite plural over the negative quantifier would free the bound variable contained in it. Thus, the definite is unsavably stuck in a downward-entailing context and cannot trigger an implicature.

The gap that Križ & Chemla found for sentences like (111) was less pronounced than in other contexts, but subjects did answer *neither* at a rate of 25%. This is a comparatively small, but robust effect that the implicature theory is at pains to explain. The only option would seem to be this: to postulate that local exhaustification under negation is possible, and that a gap judgment arises when a sentence has two parses with conflicting truth values.

(112) a. No boy found any of his presents.
b. No boy EXH found his presents.
= No boy found all of his presents.

This is rather unconvincing. First, local exhaustification is a contentious idea in itself,[36] but local exhaustification in downward-entailing enviroments is held to be impossible[37] even by those who allow for local implicatures in other contexts. Second, it is now a mystery who no appreciable number of subjects gave responses on the basis of the locally exhaustified reading alone. If they had done so, they should have answered *completely true* in at least some fraction of cases. In fact, however, the rate of such answers is so low that one cannot assume it to be anything but noise. It seems that some additional stipulation would be needed to ensure that a sentence never counts as true if only its locally exhaustified reading is true.

Such a stipulation, however, is very much at odds with the behaviour of definite plurals in restrictors. Local exhaustification in a downward-entailing context is the only way in which the implicature theory can explain the meaning of (113), where the definite plural in the relative clause has a universal meaning.

36 Opponents of the idea are Russell (2006, 2012), Geurts (2009), Geurts & Pouscoulous (2009), Geurts & van Tiel (2012), and van Tiel (2013). Local exhaustification is frequently used in the grammaticalist tradition and has been specifically defended on the basis of experimental evidence by Clifton & Dube (2010) and Chemla & Spector (2011, 2014).

37 At least without strong focal stress on the scalar item, which in this case is impossible because the contrasting part of the definite article is, effectively, the empty string, since *the* is contained in *some of the*.

(113) Every student who solved the math problems passed the exam.

1.6.3 *Processing Data*

Traditional scalar implicatures have been found to lead to slower response times (see Bott & Noveck 2004 and followers); that is to say, responses that take into account the scalar implicatures take longer than those that are based just on literal sentence meaning. This makes intuitive sense: the implicature takes time to compute.

In light of this, it would seem that Magri's theory predicts that judgements should take longer when a definite plural is understood as a universal than when it is existential. This, however, is the opposite of what Schwarz (2013) actually founds. Subjects were asked to judge sentences like (114) with respect to a display consisting of coloured geometric shapes.

(114) The circles are green.

When some, but not all of the shapes were of the target colour, subjects still judged the sentence true in a sizeable proportion of cases, but, crucially, took longer to do so than it took them to reject it. But it is rejection that requires the universal reading.

This poses a challenge to the conceptual underpinnings of Magri's theory. It must be noted, however, that free choice effects, for which an explanation in terms of double exhaustification has been proposed (Fox 2007), show the same behaviour and violate the expectation that, as implicatures, they should increase processing times (Chemla & Bott 2014). Thus, to the extent that one is convinced of the implicature analysis of free choice effects, one need not necessarily consider Schwarz's data a strong argument against Magri's approach.

1.6.4 *Finer-Grained Contexts*

There are contexts where finer distinctions matter than just the difference between *some* and *all*. In these contexts, *many* is arguably a relevant alternative of *some*, and so the implicature in (115) arises. Assume, for example, that a team of researchers performing a sleep study is impatiently waiting for their subjects to fall asleep. Here the precise numbers may be considered to be relevant because the researchers care about how much longer they have to wait. In such a context, it seems that an implicature from *some* to *not many*, or at least to *not most*, arises.

(115) Some of the subjects are asleep already.
$\rightsquigarrow$ Not many of them are.

If this is correct, then Magri's theory predicts that in such contexts, a definite plural ends up meaning *many or all* (or *most of all*) instead of *all*. It is true that definite plurals sometimes seem to mean *many or all*—this phenomenon will be the subject of chapter 3. But as will become clear, this happens precisely *not* in those contexts where finer distinctions in number are relevant. In the sleep study scenario, for example, we would surely judge (116) true only if all of the subjects are asleep, and not as soon as most of them are.[38]

(116) The subjects are asleep.

1.6.5 *Collective Predicates*

Since both *some* and *all* can take a collective predicate in their scope, the theory makes predictions for those, too. In particular, (117a) should mean that the play was performed by all and only the boys, and (117b) should mean that no subgroup of the boys performed the play.

(117) a. The boys performed *Hamlet*.
b. The boys didn't perform *Hamlet*

Note that this presupposes that *all* is compatible with such predicates (as well as with cumulative readings); otherwise, since the *all*-alternative is not available, a definite plural with such a predicate should receive only an existential reading. This means that speakers who do not accept *all* with such predicates pose a problem for the theory: why are those predicate homogeneous nonetheless?

What the theory also does not, and cannot, capture in any case is upward homogeneity with collective predicates. In a situation where the boys and the girls together performed *Hamlet*, both (118a) and (118b) are not true.

(118) a. Some of the boys performed *Hamlet*.
b. All of the boys performed *Hamlet*.

This means that (119a) is predicted to be unambiguously false, and its negation (119b) should be unambiguously true.

(119) a. The boys performed *Hamlet*.
b. The boys didn't perform *Hamlet*.

1.6.6 *The Definite as the Source of Homogeneity*

The implicature theory takes the source of homogeneity to be located effectively in the definite article, or at least the definite plural as a syntactic constituent, and not in the predicate. This raises a number of problems.

1.6.6.1 *Non-Homogeneous Predicates*

The existence of non-homogeneous predicates becomes a source of trouble. Both *some* and *all* are incompatible with non-homogeneous predicates such as *numerous* and *few in number*. It is rather unclear why they are, when (120a), which is supposed to have the same literal meaning as (120b), is perfectly acceptable.

38 The sleep study scenario is from Lasersohn 1999 and will be discussed in some greater detail in section 3.1.2.

(120) a. The boys are numerous.
b. #Some of the boys are numerous.
c. #All of the boys are numerous.

It is also unclear how, given that the definite is supposed to be existential, (121a) comes to mean what it means. Of course one can always choose a subgroup of the boys that is so small as to be *few in number*, so on an existential reading, (121a) should just be a tautology. The universality of the definite comes from an implicature, which would be computed on the basis of (121b) and (121c). Only those happen to be impossible sentence. How could (121a) possibly have an implicature based on alternatives that are impossible sentences?

(121) a. The boys are few in number.
b. #Some of the boys are few in number.
c. #All of the boys are few in number.

And even if that were somehow miraculously possible, one would still run into problems with non-homogeneous predicates that are not monotonic with respect to individual parthood. Imagine a situation where there are, in fact, ten boys. Then of course there is some group of boys that has five members, and so on an existential reading, (122a) should be true. But of course all boys taken together make for a group with more than five members, so the universal alternative is false. According to Magri's theory, then, (122) should fail to be either true or false in the ordinary way of homogeneity violations. In truth, however, it is plainly false.

(122) Die Buben sind zu fünft.
the boys are TO 5-T
'The boys are a group of five.'

This fact, the theory cannot explain.

1.6.6.2 *Generalisability*

According to the implicature theory, homogeneity can only arise when suitable alternatives are available. But the problem is that homogeneity can be found also when there is no definite article present; for example, with proper names of atomic objects (for example, book titles) when the homogeneity is with respect to a different notion of parthood.

(123) *Decline and Fall* is intelligently written.

It cannot possibly be the case that there is a lexical scale of alternatives comprised of *Decline and Fall* and *part of Decline and Fall*. The only way to make sense of this is for *part of* to be a lexical alternative of some phonologically null existential quantifier. This would mean that, in fact, *every* predication is mediated by a silent existential quantifier for the relevant domain. It is then this silent quantifier that has as an alternative an overt existential quantifier, which in turn has an overt universal as another alternative. That, however, seems very strange: why should that silent existential be there in the first? There is no particular reason for that. And even so, this works only as long as a corresponding overt existential and universal are available in the lexicon. This may be the case for homogeneity with respect to non-individual parts and kinds, but it is very doubtful for conditionals and embedded questions.

1.6.6.3 *Adverbial Quantifiers and Pronoun Binding*

The implicature theory, as stated, forces one to assume a rather curious meaning for adverbial quantifiers. Since the definite *the boys* has an existential meaning, the predicate *all came* cannot just take an individual as its argument. Rather, it must take as its argument an existential quantifier, extract from it the restrictor individual, and then apply the predicate *came* to it.[39] Analogous extraction of the restrictor has to be performed by all adverbial quantifiers.

(124) The boys all came.

There is, however, a further problem that can be overcome by no such trick. The definite plural doesn't function like an existential when it comes to binding possessive pronouns. (125) clearly presupposes that every cat has an owner; perhaps even that all the cats are owned by the same person.

(125) Peter didn't return the cats to their owner.

However, presuppositions do not usually project universally from under existential quantifiers (Chemla 2009). To see this clearly, compare (125) with (126), which does not presuppose with anything like the same force that every cat has an owner, let alone that they all have the same owner.

(126) Peter didn't return any cats to their owner.

1.6.7 *Interim Summary*

The above discussion has revealed a host of smaller and larger dissonances in the implicature theory of homogeneity. Chief among them are the fact that homogeneity projects from downward-entailing contexts, which should make implicatures disappear, its inability to account for upward homogeneity, and the problems caused by the identification of the definite plural, as a syntactic constituent, as the source of homogeneity: this makes it unclear how non-homogeneous predicates are possible and can have the meaning they have and prevents the theory from being suitably generalised to the various instances of homogeneity that do not involve a definite plural. Adverbial quantifiers also don't really fit into the picture. In light of these considerations, I conclude that this approach is on the wrong track.

39 Incidentally, this is also what *all of* must do in the compositional semantics in order to obtain the right meaning for *all of the boys*. Extracting the restrictor individual from an existential quantifier can be done by applying to it the function $\lambda Q.\iota x.Q(\lambda y.x = y)$.

1.7 THE NATURE OF HOMOGENEITY

1.7.1 *Homogeneity as a Presupposition*

Homogeneity has frequently been called a presupposition (Schwarzschild 1994, Löbner 2000, Gajewski 2005). I believe this to be mostly a historical accident in that presuppositions have always been tha paradigmatic case of a truth value gap. To my knowledge, no author has ever made a substantive commitment to homogeneity being a presupposition, or built a theory specifically on this assumption. It has also never been undertaken to scrutinise the categorisation of homogeneity as a presupposition by comparing its exact behaviour to that of familiar exemplars of the category.

Unfortunately, we do not have sufficient knowledge of the precise projection behaviour of presuppositions from the scope of quantifiers to make detailed comparison with homogeneity. It should be noted, however, that George (2008b,c,a) has presented a family of theories of presupposition projection, some of which predict the pattern of projection from the scope of quantifiers that was found for homogeneity.[40] George has also, not implausibly, suggested that presuppositions do not project from restrictors, as I have argued for homogeneity. One particular variant of George's theory captures this, but, ironically, predicts, a projection pattern in other areas that conflicts with what was found for homogeneity — and does so in virtue of the very mechanism that allows it to capture non-projection from restrictors.[41]

As pointed out in section 1.5.4, definite plurals sometimes seem to receive existential readings in downward-entailing restrictors. The analogous phenomenon for presuppositions would be that (127a) has a reading on which it means (127b).

(127) a. Every student who stopped smoking was rewarded.
b. Every student who either stopped smoking or never smoked was rewarded.

Such a thing doesn't seem to exist. The explanation I will give in chapter 3 for the case of definite plurals is incompatible with homogeneity being a presupposition. Thus, if homogeneity is to be treated as a presupposition, a new explanation for this situation has to be found.

Spector (2013) points out that a weak objection to analysing homogeneity as a presupposition is that violations of it cannot be objected to in the same way as presupposition failures.

(128) A: Does John know that Mary either bought all the jewels or none of them?

40 See section 2.3.1 for further discussion of this logic.

41 In particular, it is predicted that (i) is false if one boy found all of his presents and the rest found only some of them (cf. George 2008a: 367f.).

(i) The boys found their presents.

However, Križ & Chemla (2015) found that the sentence is, in fact, undefined in such a scenario.

B: Wait a minute! I didn't know she can't have bought just some of them.

(129) A: Did Mary buy the jewels?
B: #Wait a minute! I didn't know that she can't have bought just some of them.

This objection is weak insofar as we know that presuppositions differ in how easily they can be accommodated and the homogeneity presupposition might, for some reason, be extraordinarily easy to accommodate.[42]

In addition, Spector notes that it is not even clear that asking the question (129a) commits the speaker to the belief that Mary bought either all or none of the jewels. If it does not, then homogeneity does not project from questions in the first place. However, one does also find what looks like local accommodation of presuppositions in questions. If Bill is behaving very nervously all the time, one might ask (130) even if one is ignorant about whether Bill used to smoke. Thus, it might be that homogeneity is just particularly easy to accommodate locally.

(130) Did Bill (just) stop smoking (or something)?

There is one further context in which presuppositions and homogeneity seem to not behave quite alike. As is well-known, presuppositions project from the antecedent of a conditional.[43] (131), for example, presupposes that John bought the ring.

(131) If Mary knows that John bought the ring, he's probably angry.

If homogeneity were the presupposition that either every or no member of the plurality fulfills the predicate, then we would expect (132) to entail that either all or no subject is asleep. This is quite clearly not the case.[44]

(132) If the subjects are asleep, the study can start.

While local accommodation in the antecedent of a conditional is sometimes a possibility for presuppositions,[45] the fact that it is so much more natural, indeed in most cases strongly preferred, with homogeneity poses another obstacle for identifying homogeneity as a presupposition.

A final reason for doubt is that the answers given in the face of a homogeneity violation don't intuitively seem to convey quite the same thing as those with

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42 It is not clear to which extent presupposition triggers differ in their propensity towards local accommodation, but Smith & Hall 2011 have presented evidence that they do.

43 This, of course, is at odds with the idea that conditional antecedents are restrictors of some kind and that presuppositions don't project from restrictors.

44 We should point out that there seems to be an optional reading that might be explained by some sort of homogeneity projection, but it looks entirely different from what should happen if homogeneity were a presupposition. If the study is not performed on all subjects at the same time, but individually, then (132) could be used to convey that every subject is such that if it is asleep, the study can begin.

45 Consider, for example, (i), which may be uttered by a speaker trying to make sense of John's conduct even when she doesn't know whether John used to smoke.

(i) If John just stopped smoking, that would explain his erratic behaviour.

presupposition failures, even if their linguistic form may be similar. Intuitively, what B does in (133) is to acknowledge that A has gotten things at least partly right. Additionally, B also has the option of say *weeell* and appearing pensive, as if trying to decide whether to count A's utterance as true or not.

(133) A: Adam has written the books.
B: Well, have of them (anyway).
B: Weeell. . .

It is not clear that quite the same thing happens in the face of a presupposition failure. B's reply in (134) strikes me not as giving credit to A for getting things half right, but rather implying that it's somehow irrelevant that the presupposition of A's statement is not fulfilled, because what's really relevant is only whether Adam smokes now. In (133), there is no such implication that what's relevant is only whether Adam wrote half of the books. Furthermore, the pensive reaction doesn't seem appropriate for a presupposition failure.

(134) Context. *Adam has never smoked.*
A: Adam has stopped smoking.
B: Well, he doesn't smoke NOW (anyway).
B: #Weeell. . .

1.7.2 *Homogeneity and Vagueness*

The idea that there is some vagueness about plural predication is an old one, going back to at least Scha 1981, and recently the idea has emerged that homogeneity violations are akin to borderline cases of vague predicates.[46] I find this view appealing, not least because kinds of extension gaps should not be multiplied beyond necessity and presuppositions don't quite seem to fit.

I find this view intuitively appealing, in that both homogeneity violations and borderline cases of vagueness have a flavour of underdetermination about them, rather than the feeling that some preconditions for a speech act as a whole are not met, as in the case of presuppositions. There are also two noteworthy parallels between homogeneity and vagueness. First, homogeneity violations and borderline cases elicit essentially the same kind of replies.

(135) A: Adam wrote the books.
B: Well, most of them.
B: Weeell. . .

(136) A: Adam is tall.
B: Well, sort of.
B: Weeell. . .

Second, and perhaps more importantly, vague and non-vague scalar adjectives have analogues to homogeneity removers and quantifiers in general, which don't

46 This was discussed by Benjamin Spector (p. c.). While not explicit there, the idea is also only a small step away from Burnett 2012.

exist for presuppositions. In particular, *definitely* functions very much like *all* in that it modifies a vague adjective so that it becomes simply false of a borderline case

There are, however, also two ways in which homogeneity is starkly different from familiar examples of vague predication. First, Alxatib & Pelletier (2011) and Ripley (2011) have found that speakers accept apparently contradictory sentence, affirming and denying the same vague predicate of an individual, when borderline cases are concerned. An example of such is (137a). However, it is completely impossible to say (137b) in a situation where half of the books are in Dutch.

(137) a. Bill is both tall and not tall.
b. #The books both are and aren't in Dutch.

Second, borderline cases of vague predicates can be made explicit by denying both the predicate and its negation of an individual. Again, nothing of the kind is possible with pluralities that are mixed with respect to a predicate.

(138) a. Bill is neither tall nor not tall.
b. #The books are neither in Dutch nor aren't they (in Dutch).

Thus, if homogeneity is to be the same kind of phenomenon as vagueness, something will have to be said to explain these differences. Perhaps the existence of contextually determined standards for a vague open-scale adjective like *tall* is what is responsible for the above behaviour; this is something that might not apply to definite plurals.

A final point needs to be discussed in the context of homogeneity and vagueness. The defining characteristic of vagueness is often taken to be the (potential) existence of a so-called Sorites series, which for current purposes can be defined as follows.

DEFINITION 1.9. For a predicate P, a sequence of individuals $\langle a_1, \ldots, a_n \rangle$ is a *Sorites series* iff

1. $P(a_1)$ is clearly true;
2. $P(a_n)$ is clearly false; and
3. for all i ($1 \leq i < n$), one is inclined to accept that if $P(a_i)$ is true, then so is $P(a_{i+1})$.

This, of course, leads to paradox: point 3 allows one to infer from $P(a_1)$ that $P(a_2)$, which in turn entails $P(a_3)$, and so on up to $P(a_n)$. $P(a_n)$, however, is, by assumption, clearly false. For the predicate *tall*, for example, a Sorites series would be a sequence of individuals who differ in height by one one millimeter. If a is clearly tall and b is just one millimeter shorter, then certainly b is to be called tall as well. But now imagine that the last individual in the sequence is a whole meter shorter than a: that makes them clearly not tall, and so we have the paradox: by departing from a and going in steps of one millimeter, we establish for all the individuals that they are tall, but in the end we find ourselves calling someone tall who clearly isn't.

A naive attempt to construct an analogue for plural predication would be this:

(139) a. If all of the professors smiled, then "the professors smiled" is true.
b. If none of the professors smiled, then "the professors smiled" is true.
c. If n professors smiling makes "the professors smiled" true, then $n-1$ professors smiling also makes "the professors smiled" true.

One is surely very much inclined to accept the conditional premise for tallness, as formulated in (140), but I don't see much temptation to accept (139c).

(140) If a is tall and b is one millimeter shorter than a, then b is tall too.

There may be some instances of such conditionals which are plausible: in *some* contexts, one fewer professor smiling doesn't really make a difference for current purposes and we would still accept "the professors smiled" to describe a situation in which not all of the professors smiled. However, if the theory proposed in chapter 3 is on the right track and such non-maximal uses are only a pragmatic phenomenon, to which the trivalent semantics of homogeneous plural predication is conceptually prior, then this doesn't indicate any vagueness in the semantics of homogeneous plural predication, but only, if at all, in the conditions for its usage, due to vagueness about the question of which distinctions matter for current purposes.[47]

In sum, the picture of the connection and entanglement between vagueness, homogeneity, and non-maximality is still rather muddled and a topic for further research.

1.7.3 *Interim Summary*

The fact that homogeneity violations make a sentence neither true nor false raises the question of whether this failure to have a truth value can be identified with one of the other known kinds of failure to have a definite truth value: presupposition failures and borderline cases of vague predicates. Homogeneity is differentiated from presuppositions by the ways in which it is natural and appropriate to react to a violation of it, and possibly also by its projection behaviour from embedded contexts. Furthermore, the theory of non-maximal readings of definite plurals that will be presented in chapter 3 is also incompatible with homogeneity being a presupposition (cf. specifically section 3.3.7).

Homogeneity violations do, indeed, behave very similar to vagueness in many respects (though the projection pattern for vagueness is currently unknown). However, at least certain vague predicates have been found to sometimes follow a paraconsistent logic, allowing speakers to both affirm and deny them of a borderline case simultaneously. This never happens with homogeneity violations, but it is not clear whether this isn't just a feature of a particular subclass of vague predicates: it has only been tested for vague open-scale adjectives with contextual

47 I apologise for trying the reader's patience by writing a paragraph which is likely to be comprehensible only after reading chapter 3.

standards. Furthermore, there is no clear analogue of the Sorites paradox with pluralities, which makes the categorisation of homogeneity as a type of vagueness questionable.

I conclude that the conceptual status of homogeneity is currently unclear: it doesn't fit perfectly into either the category of presuppositions or that of vagueness as they are currently conceived of, and it might constitute its own kind of failure to have a definite truth value, but it may eventually be possible to unify it with one of the other categories once they are better understood. This is an area for further research.

1.8 CONCLUSION

This chapter has introduced the central character of this dissertation, the phenomenon of *homogeneity* and aimed to give a comprehensive overview of its various aspects, some of which will be analysed in greater detail in later chapters.

I have pointed out that the phenomenon is much broader in scope than has previously been recognised. Most prior works consider it only in the context of distributive plural predication, but it can also be found with collective predicates. I have argued for a particular way of conceiving of homogeneity which correctly captures its behaviour in both cases: as a joint constraint on the positive and negative extension of predicates, in particular, the constraint that an individual in the positive extension must not overlap with an individual in the negative extension (section 1.1). I have then shown that, beyond that, homogeneity is also found outside of the individual domain, and with respect to other notions of parthood than the relation that holds between atomic individuals and pluralities comprised of them (section 1.3). It is a pervasive feature of natural language semantics and the class of predicates that do not show it is relatively limited (section 1.4).

Homogeneity interacts with quantifiers in a twofold manner: first, in a manner of speaking, quantifiers selectively remove homogeneity with respect to the argument position that they fill (section 1.2), which only becomes apparent once the homogeneity of collective predication is properly taken into account; and second, undefinedness caused by the homogeneity of a plural predication embedded under a quantifier projects according to a certain pattern, which is quite well understood (section 1.5). A formal logical treatment of these aspects is the topic of the next chapter.

Despite all this, the conceptual status of homogeneity is still relatively unclear. While attempts to explain it as rooted in scalar implicatures fail (section 1.6) and the parallels with presuppositions are imperfect, it also does not behave exactly like standard cases of vagueness (section 1.7). A more detailed exploration of these matters is a task for future research.

2 THE LOGIC OF HOMOGENEITY

This chapter is devoted to the problem of treating the phenomenon of homogeneity, as presented in the previous chapter, formally in a trivalent logic. There are, essentially, three aspects to it that need to be captured:

1. homogeneity itself as a constraint on the positive and negative extension of predicates;
2. the pattern of projection of extension gaps from under quantifiers; and
3. the selective removal of homogeneity by quantifiers.

I will develop a logic with an algebraic semantics in which all three of these together follow from a single condition on the denotations of expressions, which is generalised across domains. In particular, when applied to predicates, these constraints enforce homogeneity, and when applied to quantifiers, they yield the right pattern of projection and homogeneity removal.

I will proceed in several steps. After discussing the trivalent logics that serve as inspiration (section 2.1), I will first present a logic for distributive predication with quantification over atoms (section 2.2.1), which will then be adapted to be able to deal with collective predication and quantification over pluralities (section 2.2.2). After an excursus on how to set right the predictions for non-monotonic quantifiers (section 2.3), I will then discuss how non-homogeneous predicates can be introduced into the language in a limited fashion so as not to detract from the previously established results (section 2.4).

2.1 SOME TRIVALENT LOGICS

2.1.1 *Strong Kleene Logic*

One of many possible trivalent propositional logics is *Strong Kleene* (SK) logic (after Kleene 1952), whose truth functional connectives are defined according to the following intuitive procedure for deriving the trivalent meaning from the classical meaning: a connective yields either 0 or 1 if it yields that truth value no matter how the undefined cases among its arguments are resolved, and # otherwise. Consider, for example, disjunction: it is true of $(1, \#)$, because it is true of both bivalent "repairs": of $(1, 0)$ and of $(0, 0)$. However, it is undefined of $(0, \#)$, because it is true of $(0, 1)$ and false of $(0, 0)$. Similarly, conjunction is false of $(0, \#)$ because it is false of both $(0, 1)$ and $(0, 0)$, but undefined of $(1, \#)$ because it is true of $(1, 1)$ and false of $(1, 0)$. This idea leads to the truth tables in table 2.1.

In this system, conjunction and disjunction are commutative, and unlike perhaps for presuppositions, this is clearly appropriate for homogeneity. It is also obvious that if conjunction is false of $(0, 1)$, it should also be false of $(0, \#)$. So the

p	q	$p \vee q$	$p \wedge q$	$\neg q$
1	1	1	1	0
1	0	1	0	1
1	#	1	#	#
0	1	1	0	
0	0	0	0	
0	#	#	0	
#	1	1	#	
#	0	#	0	
#	#	#	#	

Table 2.1: Strong Kleene truth tables

only real alternative that needs to be considered is supervaluationism,[1] which differs from SK logic in that it preserves more classical tautologies. In particular, the excluded middle $p \vee \neg p$ is valid in a supervaluationist logic. In SK, on the other hand, the excluded middle can never be false, but it can be undefined (if p has the truth value #) and so it isn't a tautology. This strikes me as correct for natural language as well, where not every excluded middle statement is unquestionably true.

1 Commonly used for vagueness, cf. e.g. Fine (1975) and Keefe (2000).

(1) A: Adam either read the books or he didn't read them.
B: Well, what if he read half of the books?

The natural way for the generalisation of a trivalent propositional logic to a language with quantification is led by the relationship that is usually assumed between disjunction and conjunction, and existential and universal quantification, respective. The intuitive idea is that $\exists x.\phi(x)$ should be equivalent to $\phi(a_1) \vee \cdots \vee \phi(a_n)$, where $a_1, \ldots, a_n$ are all the individuals in the domain of quantification.[2] Consequently, $\exists x.\phi(x)$ is true if there is at least one individual in the domain of which ϕ is true, and false if ϕ is false of all individuals. If there is at least one of which ϕ is undefined, but none of which it is true, then the existential quantification is also undefined. The universal quantifier can then be defined in terms of existential quantification and negation. The result are the following predictions for the quantifiers whose projection behaviour was discussed in section 1.5.

(2) $\forall x.P(x)$ (ALL)

TRUE iff P is true of all individuals.
FALSE iff P is false of at least one individual.
UNDEFINED otherwise.

(3) $\neg\exists x.P(x)$ (NO)

TRUE iff P is false of all individuals.
FALSE iff P is true of at least one individual.
UNDEFINED otherwise.

(4) $\exists x.\exists y.x \neq y \wedge P(x) \wedge P(y) \wedge \forall z : (z \neq x \wedge z \neq y) \rightarrow \neg P(z)$ (EXACTLY TWO)

TRUE iff P is true of two individuals and false of all others.
FALSE iff P is true of three or more individuals, or P false of all except at most one individual.
UNDEFINED otherwise.

Table 2.2 assumes a model with three individuals a, b, c and lists the predictions for quantification over P for various interesting configurations. They align with what was found experimentally by Križ & Chemla (2015) except for one type of situation: when P is true of at most one individual, but true or undefined of more than two (rows three and five), then quantification with *exactly two* was judged false, but here it is predicted to be undefined. I set this point aside for now and take it up again later in section 2.3.

2 Obviously, this particular formulation presupposes that all individuals have names, and that either the domain is finite or infinitely long formulae are possible. But it is easy to overcome these limitations in stating the semantics.

$P(a)$	$P(b)$	$P(c)$	all	no	exactly two
1	1	1	1	0	0
1	1	#	#	0	#
1	#	#	#	0	#
1	#	0	0	0	#
#	#	#	#	#	#
#	0	0	0	#	0
0	0	0	0	1	0

Table 2.2: Homogeneity projection in Strong Kleene logic

2.1.2 *A Trivalent Type Theory*

The intuition behind SK logic can be generalised across the type hierarchy in the following way, which closely follows Lepage 1992.[3]

DEFINITION 2.1. The relation $\preceq$ is defined as follows:
1. For all $x \in \{0, 1, \#\}$, $x \preceq x$ and $x \preceq \#$.
2. For all $f, g \in B^A$: $f \preceq g$ iff for all $x \in A$, $f(x) \preceq g(x)$.

We write $x \oplus y$ for the supremum of x and y. If $x \preceq y$, then we will call x a $\preceq$*-part* of y. The set of $\preceq$-minimal elements of A is written $AT(A)$ and called the set of *atoms* of A.

To give a few examples, $f \preceq g$ below, but not $f \preceq h$.

$$f = \begin{bmatrix} a \mapsto 1 \\ b \mapsto 0 \\ c \mapsto \# \end{bmatrix} g = \begin{bmatrix} a \mapsto 1 \\ b \mapsto \# \\ c \mapsto \# \end{bmatrix} h = \begin{bmatrix} a \mapsto 1 \\ b \mapsto 1 \\ c \mapsto \# \end{bmatrix}$$

Now all the functional domains in the logic are restricted to functions that are (upward-)monotonic with respect to the order $\preceq$.

DEFINITION 2.2. The *ontology* based on a set of E is defined as follows.
1. The domain of individuals D_e is E.
2. The domain of truth values D_t is $\{0, 1, \#\}$.
3. For any type $\sigma\tau$, the domain $D_{\sigma\tau}$ is the set of all $\preceq$-monotonic functions from D_σ to D_τ.

Lepage shows that the functional domains are then join-semilattices. They are, however, not mereological structures, because not all non-atomic functions are uniquely decomposable into atoms. The function f below, for example, is

3 Here and in all discussion to follow, I ignore functions whose type does not end in t, because they are not relevant to the questions at hand.

both $g \oplus g'$ and $h \oplus h'$. g, g', h, and h' are all atoms, and so f is not uniquely decomposable.

$$f = \begin{bmatrix} a \mapsto \# \\ b \mapsto \# \\ c \mapsto \# \end{bmatrix} g = \begin{bmatrix} a \mapsto 1 \\ b \mapsto 1 \\ c \mapsto 0 \end{bmatrix} g' = \begin{bmatrix} a \mapsto 0 \\ b \mapsto 0 \\ c \mapsto 1 \end{bmatrix} h = \begin{bmatrix} a \mapsto 1 \\ b \mapsto 0 \\ c \mapsto 0 \end{bmatrix}$$
$$h' = \begin{bmatrix} a \mapsto 0 \\ b \mapsto 1 \\ c \mapsto 1 \end{bmatrix}$$

One central feature of SK logic is already ensured now, which is that a function cannot be true or false of a list of arguments unless it is true for all ways of replacing # with 0 or 1 in those arguments.

FACT 2.1. *$f(x) = 1$ only if for all $x' \prec x$, $f(x') = 1$, and analogously for 0.*[4]

But this still allows for a lot of undefinedness: a binary boolean function could, for example, be simply undefined of $(0, 0)$. One would like to have the other direction of the conditional in fact 2.1 as well: if f is true (or false) of all parts of x, then it is also true (or false) of x. This can be effected by restricting functions to be atomic, i. e. $\preceq$-minimal, among the $\preceq$-monotonic functions.[5]

FACT 2.2. *If f is atomic, then for all non-atomic x, $f(x) = 1$ if for all $x' \prec x$, $f(x') = 1$ and analogously for 0.*

The projection behaviour of a quantifier can now be derived from its bivalent meaning, in that given a quantifier $\mathcal{Q}$ defined on bivalent predicates, there is then a unique atomic $\preceq$-monotonic quantifier $\mathcal{P}$ defined on trivalent predicates that assumes the same values as $\mathcal{Q}$ on bivalent predicates. The intuitive rule is this: in generalising $\mathcal{Q}$ to a trivalent predicate P, take all the atomic $\preceq$-parts of P, i. e. the set of functions that are obtained by replacing all instances of # in P with either 0 or 1. If $\mathcal{Q}$ is true of all these predicates, then it is also true of P; if it is false, it is false of P. Otherwise, the result is #. This is very much a sensible generalisation of the intuition behind propositional SK logic.[6] This system replicates the behaviour of quantifiers shown in table 2.2.

2.2 INTRODUCING PLURALITIES

2.2.1 *A Logic for Homogeneous Distributive Predication*

The type theory just presented can be turned into an elegant system for distributive plural predication by taking as the domain of individuals an atomic boolean algebra with the bottom element removed (as is standard since Link 1983) and

4 Of course, all implicit quantification here and in the following is restricted to the domains of the ontology; in this particular case, the set of $\preceq$-monotonic functions.

5 Not *all* functions in the model can be required to be $\preceq$-minimal, or else a function that takes an individual as its argument could never yield the value #. This slight unprincipledness will disappear once pluralities are properly integrated into the system.

6 For more discussion of how this procedure plays out in individual cases, see section 1.5.

identifying $\preceq$ on it with mereological parthood.[7] The monotonicity requirement then automatically causes all predicates to be distributive and homogeneous. Since higher types are not actually needed for present purposes, I will restrict myself to presenting a fragment with generalised quantification (cf. Barwise & Cooper 1981) instead of a full type theory.

DEFINITION 2.3. The alphabet of the language of generalised atomic quantification $\mathcal{L}_{DGQ}$ consists of:

1. constant symbols $a, b, \ldots,$
2. variables $x, y, z, x', \ldots,$
3. relation symbols $P, Q, R, P', \ldots$ with an associated arity,
4. determiner symbols $\mathcal{D}, \mathcal{D}', \ldots,$
5. parentheses (and),
6. the λ-operator,
7. the distributivity operator D, and
8. connectives $\wedge, \vee, \neg, \rightarrow$.

DEFINITION 2.4. (Syntax) One can define the syntax of this fragment of type theory[8] using two auxiliary notions. The first is that of a *relational expression*.

1. Every n-ary relation symbol is an n-ary relational expression.
2. If x is a variable and α an n-ary relational expression, then $(\lambda x.\alpha)$ is an $(n+1)$-ary relational expression. If ϕ is a formula, then $(\lambda x.\phi)$ is a unary relational expression.
3. If α is a unary relational expression, then $D(\alpha)$ is also a unary relational expression.
4. If α is an n-ary relational expression ($n > 0$) and β is a variable or constant, symbol, then $\alpha(\beta)$ is an $(n-1)$-ary relational expression.

The second auxiliary notion is that of a *quantifier*.

1. If α is a determiner symbol and β is a unary relational expression, then $\alpha(\beta)$ is a quantifier.

The set of formulae of the language $\mathcal{L}_{DGQ}$ is then defined by the following rules:

1. Every nullary relational expression is a formula.
2. If α is a quantifier and β is a unary relational expression, then $\alpha(\beta)$ is a formula.
3. The usual clauses for negation, conjunction, etc.

As a notational convention, I will often write $\alpha(\beta, \gamma)$ instead of $\alpha(\beta)(\gamma)$.

DEFINITION 2.5. A model M is a tuple $(E, \preceq, \mathcal{I})$, where $(E, \preceq)$ is a atomic boolean algebra without the bottom element and $\mathcal{I}$ is an admissible interpretation for the non-logical vocabulary.

The relation $\preceq$ is generalised to other domains as in definition 2.1.

7 This move is also made by Schmitt (2012b) for plurality-related, but different purposes.

8 The fragment is essentially a standard one-sorted type theory without the quantifiers the $\forall$ and $\exists$ and identity, restricted to expressions of type $\mathtt{e}^n\mathtt{t}$ (relational expressions), `(et)((et)t)` (determiners), `(et)t` (quantifiers), and `t` (formulae). The distributivity operator D, which is added to the logical vocabulary, is the only expression that corresponds to something of type `(et)(et)`.

DEFINITION 2.6.

1. The domain of individuals D_e is E.
2. The domain of truth values D_t is $\{0, 1, \#\}$.
3. The domain of unary relations D_{R_1} is the set of all $\preceq$-monotonic functions from D_e to D_t.
4. The domain of n-ary relations D_{R_n} is the set of all $\preceq$-monotonic functions from D_e to $D_{R_{n-1}}$.
5. The domain of determiners D_D is the set of all $\preceq$-monotonic functions from D_{R_1} to $D_t^{D_{R_1}}$

DEFINITION 2.7. An *admissible interpretation* is a function such that

1. for all constants c, $\mathcal{I}(c) \in D_e$,
2. for all n-ary relation symbols R, $\mathcal{I}(R) \in AT(D_{R_n})$, and
3. for determiner symbols $\mathcal{D}$, $\mathcal{I}(\mathcal{D}) \in AT(D_D)$.

Note that it is important for the atomicity requirement to be part of the definition of the interpretation function and not a property of the domains. The result of the application of a binary function to its first argument is, after all, not necessarily atomic: a predicate like *read the books* can be undefined of atomic individuals, for example those who read only half of the books, in which case it is not atomic. However, the value of an atomic function applied to an atom is always itself atomic.

DEFINITION 2.8. We call a formula ϕ *true* with respect to a model M and a variable assignment g iff $[\![\phi]\!]^{M,g} = 1$ and *false* iff $[\![\phi]\!]^{M,g} = 0$. $[\![\cdot]\!]^{M,g}$ is defined as follows:

1. If α is a variable, then $[\![\alpha]\!]^{M,g} = g(\alpha)$.
2. If α is a constant, a relation symbol, or a determiner symbol, then $[\![c]\!]^{M,g} = \mathcal{I}(\alpha)$.
3. $[\![\lambda x.\phi]\!]^{M,g}$ is that function which maps any individual u to $[\![\phi]\!]^{M,g[u/x]}$.
4. $[\![D]\!]^{M,g}$ maps any $f \in D_{R_1}$ to $f' : x \mapsto \bigoplus_{x' \preceq_{AT} x} f(x')$.
5. $[\![\alpha(\beta)]\!]^{M,g} = [\![\alpha]\!]([\![\beta]\!])$.
6. Negation and the boolean connectives have their SK meanings.

FACT 2.3. *For any $f \in D_{R_1}$, $[\![D]\!](f) \in D_{R_1}$.*

FACT 2.4. *All relations (denotations of relational expressions) are distributive (by $\preceq$-monotonicity) and closed under mereological fusion (by $\preceq$-minimality) wrt all argument positions. Formally: $f(x_1, \ldots, x_i, \ldots, x_n) = 1$ iff $f(x_1, \ldots, x', \ldots, x_n) = 1$ for all $x' \preceq x_i$.*

It follows straightforwardly that all relations in this system are homogeneous. Since $\preceq$ has been generalised across domains, the same can now be said of quantifiers. The discussion of quantifiers below will assume that they are the result of the application of a determiner to an atomic restrictor argument, so that the quantifier itself is also atomic.[9] Then the results about the behaviour of quantifiers (facts 2.1 and 2.2) carry over from section 2.1.2. With respect to

9 For more on restrictors, see section 2.4.4.

homogeneity projection from a plurality-denoting expression in their scope, quantifiers then behave as already discussed in the previous section. Furthermore, they remove downward homogeneity as they should: this follows directly from the conjunction of these two facts together with atomicity.

FACT 2.5. *For all predicates P and quantifiers $\mathcal{Q}$, if P is atomic, then $\mathcal{Q}(P) \neq \#$.*[10]

Proof. Assume that there were a predicate P and quantifier $\mathcal{Q}$ such that for all atomic x, $P(x) \neq \#$ and $\mathcal{Q}(P) = \#$. Then there is a $\mathcal{P}$ just like $\mathcal{Q}$ except that $\mathcal{P}(P) \neq \#$. By definition of $\preceq$, $\mathcal{P} \prec \mathcal{Q}$. $\mathcal{P}$ is also monotonic. Assume that $\mathcal{P}$ is not monotonic. Since P is atomic, this can only be so if there is a $P' \succ P$ such that $\mathcal{P}(P') = \neg\mathcal{P}(P)$ (i. e. the one is 1 and the other is 0). But since $\mathcal{Q}$ and $\mathcal{P}$ agree on all predicates except P, it follows that $\mathcal{Q}(P') \neq \#$. Then $\mathcal{Q}$ is not monotonic and not a possible quantifier. So if $\mathcal{Q}$ is monotonic, it is not atomic because $\mathcal{P}$ is also monotonic. Hence there is no such possible quantifier $\mathcal{Q}$. □

Let this be illustrated with the case of *all*. Let $\mathcal{Q}$ correspond to *all boys* and *a* and *b* be boys. The quantifier is true of all ($\preceq$-monotonic, because others aren't permitted) predicates that are true of $a \oplus b$. We want *all* to be undefined of Q and false of Q'. What we can show is that this follows from the fact that $\mathcal{Q}(P) = 1$ together with atomicity and $\preceq$-monotonicity.

$$P = \begin{bmatrix} a & \mapsto 1 \\ b & \mapsto 1 \\ a \oplus b & \mapsto 1 \end{bmatrix} Q = \begin{bmatrix} a & \mapsto 1 \\ b & \mapsto \# \\ a \oplus b & \mapsto \# \end{bmatrix} Q' = \begin{bmatrix} a & \mapsto 1 \\ b & \mapsto 0 \\ a \oplus b & \mapsto \# \end{bmatrix}$$

By assumption, $\mathcal{Q}$ is not true of Q and Q' because those aren't true of $a \oplus b$. Since $P \prec Q$, $\mathcal{Q}(Q)$ cannot be 0 on pain of violating monotonicity, and so $\mathcal{Q}(Q) = \#$. Q', however, is atomic and there is no $Q'' \succ Q'$ such that $Q''(a \oplus b) = 1$ (because such a Q'' would violate monotonicity). It is therefore safe to have $\mathcal{Q}$ be false of Q', and by $\preceq$-minimality, it *must* then be false.

An elegant feature of this system is that the homogeneity of predicates and the projection properties of quantifiers are governed by a single constraint that applies across domains; but of course it covers only a rather restricted fragment of natural language.

2.2.2 *Collective Predication and Plural Quantification*

The next thing we would, of course, like to add to the language is collective predicates. The clauses for the syntax and the denotation function $[\![\cdot]\!]^{M,g}$ are identical to those for the language of the previous section. But the conditions imposed on the domains are too strong and have to be replaced by something weaker. The obvious thing to do is to take the actual formulation of the homogeneity constraint from chapter 1 and state it formally in such a manner as to be applicable across domains. For unary predicates, the constraint was this: no

10 From here on, I will frequently use P, Q, etc. for both predicate symbols and their denotations (i. e. functions) in cases where no confusion is likely to arise.

individual in its positive extension must overlap with an individual in its negative extension. This can be captured by the following definition.

DEFINITION 2.9. (Homogeneity) Let $\between_A$ denote overlap wrt the relation $\preceq$ restricted to the set A; that is to say, $x \between_A y$ if and only if there is a $z \in A$ such that $z \preceq x$ and $z \preceq y$. A function f is *homogeneous* iff for all x, y, $x \between_{\text{dom}(f)} y \rightarrow f(x) \between_{\text{range}(f)} f(y)$.[11]

The functional domains of the model are now restricted to be homogeneous.

DEFINITION 2.10.

1. The domain of individuals D_e is E.
2. The domain of truth values D_t is $\{0, 1, \#\}$.
3. The domain of unary relations D_{R_1} is the set of all homogeneous functions from D_e to D_t.
4. The domain of n-ary relations D_{R_n} is the set of all homogeneous functions from D_e to $D_{R_{n-1}}$.
5. The domain of determiners D_D is the set of all homogeneous functions from D_{R_1} to $D_t^{D_{R_1}}$

Note that we are operating here with $\preceq$ restricted to the lower domains in assessing homogeneity of a higher domain. This is analogous to what was done for monotonicity before. Thus, two functions only overlap, for the purposes of homogeneity, only if they have a part in common that is itself homogeneous. f and g below do not overlap, because h, which is their only common $\preceq$-part, is not homogeneous and so not in the domain. This will turn out to be crucial.

$$f = \begin{bmatrix} a & \mapsto \# \\ b & \mapsto \# \\ a \oplus b & \mapsto 1 \end{bmatrix} g = \begin{bmatrix} a & \mapsto 1 \\ b & \mapsto 0 \\ a \oplus b & \mapsto \# \end{bmatrix} h = \begin{bmatrix} a & \mapsto 1 \\ b & \mapsto 0 \\ a \oplus b & \mapsto 1 \end{bmatrix}$$

To see how this enforces exactly the desired non-overlap constraint between the positive and negative extension of predicates, consider this. If x and y overlap, then it cannot be the case that $f(x) = 1$ and $f(y) = 0$, because 1 and 0 don't have a part in common. Hence, the truth values of f for overlapping x and y have to be either the same, or one of them must be #.

Obviously now that we have collective predicates, we cannot require all denotations of relation symbols to be $\preceq$-minimal — that would make them distributive. So right now predicates are allowed to be undefined wherever they want, which isn't what we observe in natural language: predicates are only undefined in those places where homogeneity forces them to be. Given their positive extension, they are false of all individuals they can be false of without violating homogeneity. This can be captured by atomicity with respect to a different order.

DEFINITION 2.11. The order $\preceq_0$ is defined as follows.

1. For all $x \in \{0, 1, \#\}$, $x \preceq_0 x$ and $0 \preceq_0 \#$.
2. For all $f, g \in B^A$: $f \preceq_0 g$ iff for all $x \in A$, $f(x) \preceq_0 g(x)$.

11 I will ommit the subscript in the following whenever the relevant set is obvious.

We write $AT_0(A)$ for the $\preceq_0$-minimal elements of a set A.

$f \preceq_0 g$ if and only if f is true of exactly the same individuals that g is true of and false of at least all the individuals that g is false of, i. e. $\{x \mid f(x) = 1\} = \{x \mid g(x) = 1\}$ and $\{x \mid f(x) = 0\} \supseteq \{x \mid g(x) = 0\}$. This is now what we put as a constraint on the interpretation function $\mathcal{I}$ instead of $\preceq$-minimality.

DEFINITION 2.12. An *admissible interpretation* is a function such that

1. for all constants c, $\mathcal{I}(c) \in D_e$,
2. for all n-ary relation symbols R, $\mathcal{I}(R) \in AT_0(D_{R_n})$, and
3. for determiner symbols $\mathcal{D}$, $\mathcal{I}(\mathcal{D}) \in AT_0(D_D)$.

Note that unlike $\preceq$-minimality, $\preceq_0$-minimality does not entail closure under mereological fusion. This property would have to be enforced separately, if desired.

As for quantifiers, homogeneity alone does not yet ensure much in the way of the properties that we want, and the addition of $\preceq_0$-minimality is also not enough: it does not follow that quantifiers are distributive ($\mathcal{Q}(P) = 1$ entails $\mathcal{Q}(P') = 1$ for all $P' \prec P$). But there is no quantifier in natural language that is true of $P \oplus P'$, while not being true of P and P' individually. This, like the fact that quantifiers in natural language are conservative, will be regarded as an accident of lexicalisation. From distributivity, it then follows that quantifiers remove downward-homogeneity:

FACT 2.6. *For all predicates P and distributive quantifiers $\mathcal{Q}$, if P is atomic,*[12] *then* $\mathcal{Q}(P) \neq \#$.

Proof. Assume that there were a P such that for all atomic x, $P(x) \neq \#$ and $\mathcal{Q}(P) = \#$. Then there is a function $\mathcal{P}$ just like $\mathcal{Q}$ except that $\mathcal{P}(P) = 0$. By definition of $\preceq$, $\mathcal{P} \preceq \mathcal{Q}$. If $\mathcal{P}$ is homogeneous, $\mathcal{Q}$ is not $\preceq_0$-minimal in its domain and so not a possible quantifier. Assume that $\mathcal{P}$ is not homogeneous. Since P is atomic, the only way this can happen is if there is a $P' \succ P$ such that $\mathcal{P}(P') = 1$. Since $\mathcal{P}$ and $\mathcal{Q}$ agree on every predicate but P, $\mathcal{Q}(P') = 1$. But since $P \prec P'$ and $\mathcal{Q}(P) \neq 1$, $\mathcal{Q}$ is not distributive. $\square$

Collective predication is possible now and upward homogeneity is not removed. Look, for example, at a quantifier $\mathcal{Q}$ that intuitively corresponds to *two people*: it is true of all and only those predicates that are true of a duality of individuals. Hence $\mathcal{Q}(P) = 1$. At the same time, should be the case that $\mathcal{Q}(P') = \#$.

$$P = \begin{bmatrix} a & \mapsto \# \\ b & \mapsto \# \\ a \oplus b & \mapsto 1 \end{bmatrix} P' = \begin{bmatrix} a & \mapsto \# \\ b & \mapsto \# \\ a \oplus b & \mapsto \# \end{bmatrix}$$

The desired truth value assignments obviously do not cause a violation of $\preceq_0$-minimality, since P and P' overlap and so $\mathcal{Q}$ can't be false of P', by homogeneity.

12 Here as always, unless otherwise noted, atomicity refers to $\preceq$-atomicity, not $\preceq_0$-atomicity.

The interesting part of the situation is this: by removal of downward homogeneity, $\mathcal{Q}$ is false of P''.

$$P'' = \begin{bmatrix} a & \mapsto 1 \\ b & \mapsto 0 \\ a \oplus b & \mapsto \# \end{bmatrix}$$

We need to show that this doesn't lead to a homogeneity violation. Indeed it does not: P'' does not overlap in the relevant sense with any homogeneous predicate that is true of $a \oplus b$, because the only common parts it could have with such a predicate are non-homogeneous. P and P'' have only one common part which is Q below. But Q is not homogeneous, so it doesn't count for determining overlap between P and P'' when assessing the homogeneity of the quantifier.

$$Q = \begin{bmatrix} a & \mapsto 1 \\ b & \mapsto 0 \\ a \oplus b & \mapsto 1 \end{bmatrix}$$

Note, finally, a property which quantifiers do not necessarily have, and which would ruin the correct treatment of upward homogeneity.

DEFINITION 2.13. (Inverse Distributivity) A function f is *inversely distributive* if $f(x) = 1$ if for all $x' \prec x$, $f(x') = 1$.

Take the quantifier $\mathcal{Q}$ that corresponds to *a student*, which is true of every predicate that is true of an atomic student. Assume furthermore that a and b are both students. Then $\mathcal{Q}$ is true of P, P', and P'', but it should be undefined, by upward homogeneity, of Q. But P, P' and P'' are all the (homogeneous, but only those count) proper $\preceq$-parts of Q, and so $\mathcal{Q}$ is not inversely distributive.

$$P = \begin{bmatrix} a & \mapsto 1 \\ b & \mapsto \# \\ a \oplus b & \mapsto 1 \end{bmatrix} P' = \begin{bmatrix} a & \mapsto \# \\ b & \mapsto 1 \\ a \oplus b & \mapsto 1 \end{bmatrix} P'' = \begin{bmatrix} a & \mapsto 1 \\ b & \mapsto 1 \\ a \oplus b & \mapsto 1 \end{bmatrix}$$

$$Q = \begin{bmatrix} a & \mapsto \# \\ b & \mapsto \# \\ a \oplus b & \mapsto 1 \end{bmatrix}$$

It is, of course, possible for a quantifier to be true of all four of these predicates; *all of* $a \oplus b$, for example, is. These are just different quantifiers and don't mean what *a student* means.

2.3 NON-MONOTONIC QUANTIFIERS

We have seen that the predictions of the systems discussed above regarding homogeneity projection from the scope of non-monotonic quantifiers are mostly, but not quite aligned with the pattern that was found experimentally. This section presents a way of fixing this problem that is inspired by the work of Ben George on the logic of presupposition projection.

2.3.1 *George's (2008) Logics*

George (2008b,c,a) takes a different approach to generalising the intuition underlying propositional SK logic to higher orders. The handling of trivalence is not built into the definition of the functional domains, but rather into semantic composition; the ontology remains unrestricted.

DEFINITION 2.14. The ontology based on a set E is defined as follows.

1. The domain of individuals D_{e} is E.
2. The domain of truth values D_{t} is $\{0, 1, \#\}$.
3. The domain of type $\sigma\tau$ $D_{\sigma\tau}$ is the set of functions from D_σ to D_τ.

The model contains an interpretation function $\mathcal{I}$ that assigns every constant an element of the domain of its type. Everything so far is just as usual in any sort of type theory. The novelty lies in the semantic composition rules: instead of function application, a special notion of function deployment is used. The idea behind function deployment is this: when deploying a function f on an argument that contains some undefinedness, we look at all the ways in which the undefinedness in the argument can be repaired. If f yields the same value for all such repairs, then that is the result of the deployment; otherwise the result is itself undefined. The intuition behind the notion of a repair is an object that has some undefinedness about it (either the third truth value itself, or a function that maps certain arguments to #) is repaired by putting in 0 or 1 instead of all the #.

DEFINITION 2.15. (Repairs)

1. The repairs for truth values are given by $0^{0/\#} = 0^{1/\#} = \#^{0/\#} = 0$ and $1^{0/\#} = 1^{1/\#} = \#^{1/\#} = 1$.
2. For any n-ary function f, $f^{1/\#}$ is that relation such that for all $x_1, \ldots, x_n$, $f^{1/\#}(x_1, \ldots, x_n) = f(x_1, \ldots, x_n)^{1/\#}$, and analogously for $f^{0/\#}$.

Note that in a repair, all # need to be replaced by the same definite truth value. Thus, f below is not a repair of g.

$$f = \begin{bmatrix} a \mapsto 1 \\ b \mapsto 0 \\ c \mapsto 0 \end{bmatrix}$$

$$g = \begin{bmatrix} a \mapsto \# \\ b \mapsto \# \\ c \mapsto 0 \end{bmatrix}$$

The crucial notion of deployment is now defined as follows.

DEFINITION 2.16. (Function Deployment) The deployment of an n-ary function f on list of arguments $x_1, \ldots, x_n$ (written $f[x_1, \ldots, x_n]$) is defined as follows:

1. if there is an a such that for all repairs $y_1, \ldots, y_n$ of $x_1, \ldots, x_n$, $f(y_1, \ldots, y_n) = z$, then $f[x_1, \ldots, x_n] = a$;
2. otherwise, $f[x_1, \ldots, x_n] = \#$.

This is now what's used in the rule for semantic composition instead of function application: $[\![\alpha(\beta)]\!]^{M,g} = [\![\alpha]\!]^{M,g}[[\![\beta]\!]^{M,g}]$. The system[13] yields the same results for projection from the scope of quantifiers as that in section 2.1.2 with a minor difference: it deviates in just one particular kind of scenario involving non-monotonic quantifiers. Assume that there are only three students a, b and c, and the predicate P is as follows:

$$P = \begin{bmatrix} a \mapsto 1 \\ b \mapsto \# \\ c \mapsto \# \end{bmatrix}$$

Then *exactly two students*, deployed on P, is simply false, since it is false of both available repairs: $P^{1/\#}$ is true of three students, and $P^{0/\#}$ is true of one.

$$P^{1/\#} = \begin{bmatrix} a \mapsto 1 \\ b \mapsto 1 \\ c \mapsto 1 \end{bmatrix}$$

$$P^{0/\#} = \begin{bmatrix} a \mapsto 1 \\ b \mapsto 0 \\ c \mapsto 0 \end{bmatrix}$$

P' below, on which *exactly two students* is true, is not an acceptable repair because it resolves the two undefined cases differently.

$$P' = \begin{bmatrix} a \mapsto 1 \\ b \mapsto 1 \\ c \mapsto 0 \end{bmatrix}$$

This is in accordance with what was found by Križ & Chemla (2015) and thus a desirable feature of the logic. The system also has a very undesirable property, however: β-conversion is not valid in it. As in SK, the excluded middle $p \vee \neg p$ isn't a tautology. If p is undefined, then both disjuncts are undefined and so the whole disjunction is. But the λ-abstracted version of it, $(\lambda q.q \vee \neg q)(p)$ is a tautology. Since for both 0 and 1, the predicate $[\![\lambda q.q \vee \neg q]\!]$ yields 1, the deployment of $[\![\lambda q.q \vee \neg q]\!]$ on $[\![p]\!]$ also yields 1. This awkward feature generalises to λ-abstraction over all types that end in $\texttt{t}$. The reason for it is that the handling of trivalence is built into the composition rule, and not, as in the other systems presented in this chapter, encoded in the function denotations.

2.3.2 *An Algebraic Perspective*

This problem, however, can be solved, and the desirable prediction for non-monotonic quantifiers retained, by a switch to an algebraic perspective. All that is needed is replacement of the order $\preceq$ that was used in section 2.1.2 by something ever so slightly different:

13 What I have presented here is just the basic version. George discusses a number of adaptations of the logic to make it suited to the analysis of presupposition projection, which are not relevant for present purposes.

DEFINITION 2.17.

A. The order $\preceq_0$ is defined as follows:
   (a) For all $x \in \{0, 1, \#\}$, $x \preceq_0 x$ and $0 \preceq_0 \#$.
   (b) For all $f, g \in B^A$: $f \preceq_0 g$ iff for all $x \in A$, $f(x) \preceq_0 g(x)$.

B. The order $\preceq_1$ is defined analogously.

C. $x \preceq_{0|1} y$ iff $x \preceq_0 y$ or $x \preceq_1 y$.

$\preceq_{0|1}$ is weaker than $\preceq$: the latter is the transitive closure of the first. f is a $\preceq$-part of g if it is just like g except that some of the # in g have been replaced by 0 or 1. In order to be a $\preceq_{0|1}$-part, it also has to be the case that all these replacements take the same value, either 0 or 1 — one can't mix them.

If we now replace the requirement of $\preceq$-monotonicity with just $\preceq_{0|1}$-monotonicity, we have what we want: the desired meaning for *exactly two* isn't $\preceq$-monotonic, but it is $\preceq_{0|1}$-monotonic, because $P \npreceq_{0|1} P'$. Thus *exactly two people* can be false of P' despite being true of P.

$$P = \begin{bmatrix} a \mapsto 1 \\ b \mapsto 1 \\ c \mapsto 0 \end{bmatrix} P' = \begin{bmatrix} a \mapsto 1 \\ b \mapsto \# \\ c \mapsto \# \end{bmatrix}$$

All the other results from section 2.1.2 carry over, with $\preceq$ suitably replaced by $\preceq_{0|1}$. Since $\preceq_{0|1}$ is still identified with mereological parthood on the domain of individuals, pluralities and collective predicates as well as quantification over pluralities can be introduced in exactly the same fashion as in sections 2.2.1 and 2.2.2, since none of the arguments there rely on the transitivity of $\preceq$.

Naturally, β-conversion is now valid, since semantic composition is back to using function application.

2.4 WHAT'S MISSING

The logic for homogeneous collective predication that was defined in section 2.2.2 is very elegant, but of course it lacks a number of features that we would need in order to analyse a more complete fragment of natural language in it. Adding the desired features will require us to leave the nice walled garden we have been inhabiting so far, where there are only homogeneous functions. However, the intrusion of non-homogeneity will be limited: non-homogeneous predicates arise only through λ-abstraction over terms that contain one of a handful of members of the logical vocabulary.

2.4.1 *Cumulative Relations and Identity*

The systems discussed so far do not permit the kind of relation that is needed to capture what's called *cumulative readings* in linguistics: the reading on which (5) means that each of the students read at least one book and every book was read by at least one of the students.

(5) The students read the books.

I will call such relations *interestingly cumulative relations*.[14]

DEFINITION 2.18. (Interesting Cumulativity) An n-ary relation R is *interestingly cumulative* iff

1. there are distinct individuals a, b, c, d such that $R(a, b) = R(c, d) = 1$ and either $R(a, d) = 0$ or $R(c, b) = 0$, and
2. R is closed under pointwise fusion.

FACT 2.7. *If R is interestingly cumulative, then R is not homogeneous.*

Proof. Assume that there are a, b, c such that $R(a, b) = R(b, c) = 1$ and $R(a, c) = 0$ and R is closed under pointwise fusion (i. e. R is interestingly cumulative). Then by closure under pointwise fusion, $R(a \oplus b, b \oplus c) = 1$. By homogeneity of R, $R(a)$ and $R(a \oplus b)$ should overlap. For any common $\preceq$-part f of $R(a)$ and $R(a \oplus b)$, it must be the case that $f(c) = 0$ and $f(b \oplus c) = 1$. But then f is not homogeneous and so not in D_{R_1}, so $R(a)$ and $R(a \oplus b)$ don't overlap because they have a $\preceq$-part in common that is in D_{R_1}. Hence R is not homogeneous. □

As an example, take the relation below.

$$R = \begin{bmatrix} a & \mapsto & \begin{bmatrix} c & \mapsto 1 \\ d & \mapsto 0 \\ c \oplus d & \mapsto \# \end{bmatrix} \\ b & \mapsto & \begin{bmatrix} c & \mapsto 0 \\ d & \mapsto 1 \\ c \oplus d & \mapsto \# \end{bmatrix} \\ a \oplus b & \mapsto & \begin{bmatrix} c & \mapsto \# \\ d & \mapsto \# \\ c \oplus d & \mapsto 1 \end{bmatrix} \end{bmatrix}$$

It is interestingly cumulative, and it's not homogeneous because $R(a)$ and $R(a \oplus b)$ have no homogeneous $\preceq$-part in common. Their only common $\preceq$-part is the non-homogeneous Q:

$$Q = \begin{bmatrix} c & \mapsto 1 \\ d & \mapsto 0 \\ c \oplus d & \mapsto 1 \end{bmatrix}$$

If non-homogeneous $\preceq$-parts were taken into account for the purpose of evaluating overlap, then interestingly cumulative relations would be allowed. But the exclusion of non-homogeneous common parts from consideration is necessary to make quantification over pluralities possible.

There is also a very particular interestingly cumulative relation that we should very much like to have: identity. Of course it has to be possible that $id(a, b) = 0$ while $id(a \oplus b, a \oplus b) = 1$. A desirable homogeneous identity function

14 An uninterestingly cumulative relation would be one which is closed under mereological fusion, but not false of any pair of atoms. Such relations are allowed by the homogeneity constraint.

(cf. section 4.4.1) would look just like a cumulative relation and be inadmissible for the same reason.

$$id = \begin{bmatrix} a & \mapsto & \begin{bmatrix} a & \mapsto 1 \\ b & \mapsto 0 \\ a \oplus b & \mapsto \# \end{bmatrix} \\ b & \mapsto & \begin{bmatrix} a & \mapsto 0 \\ b & \mapsto 1 \\ a \oplus b & \mapsto \# \end{bmatrix} \\ a \oplus b & \mapsto & \begin{bmatrix} a & \mapsto \# \\ b & \mapsto \# \\ a \oplus b & \mapsto 1 \end{bmatrix} \end{bmatrix}$$

I have pointed out that this problem can be solved by applying closure under pointwise fusion *after* checking for homogeneity. This can be accomplished by requiring the denotations of constants to be homogeneous and adding a closure operator in the logical language. Then this operator can simply be a part of the translation of lexical items of natural language, so that, for example, $\|\text{read}\| = {}^*read$, for some binary predicate constant *read* of the logical language.

DEFINITION 2.19. (Closure under Pointwise Fusion)

1. (Syntax) If α is an expression of a type ending in $\mathtt{t}$, then so is ${}^*\alpha$.
2. (Semantics) $[\![{}^*\alpha]\!]$ is the closure of $[\![\alpha]\!]$ under pointwise fusion.

An identity relation with the desired properties can now also be added. I will call it *homogeneous identity* even though it is not entirely homogeneous in the technical sense, because it is still quite far from regular logical identity.

DEFINITION 2.20. (Homogeneous Identity)

1. (Syntax) If α and β are expressions of the same type, then $\alpha \doteq \beta$ is an expression of type $\mathtt{t}$.
2. (Semantics) $[\![\doteq]\!]$ is the closure under fusion of the $\preceq_0$-minimal function that agrees with logical identity on $AT(D_\sigma)$ for all types σ.[15]

Note that even a relation that is closed under pointwise fusion is still homogeneous with respect to each individual argument position when the others are kept constant.

FACT 2.8. *If f' is the closure under pointwise fusion of a homogeneous n-ary relation f, then for all i $(1 \leq i \leq n)$ and tuples $\langle x_1, \ldots, x_{n-1}\rangle$, the predicate $f'' : y \mapsto f'(\langle x_1, \ldots, x_{i-1}, y, x_i, \ldots, x_{n-1}\rangle)$ is homogeneous.*

However, we have still opened the door to defining unary non-homogeneous predicates. Even if the quantifier $\mathcal{Q}$ and the relation R are perfectly homogeneous, the relation denoted by the λ-expression in (6) may not be.

15 Equivalently:

$$[\![\alpha \doteq \beta]\!] = \begin{cases} 1 & \text{iff } [\![\alpha]\!] = [\![\beta]\!] \\ 0 & \text{iff not } [\![\alpha]\!] \between [\![\beta]\!] \\ \# & \text{otherwise} \end{cases}$$

(6) $\lambda x.\mathcal{Q}(\lambda y.R(x,y))$

To see this, assume there are two people a and b, and two books c and d. Let $[\![R]\!]$ be the closure under pointwise fusion of the relation of reading and be as follows:

$$[\![\|\text{read}\|]\!] = \begin{bmatrix} a & \mapsto \begin{bmatrix} c & \mapsto 1 \\ d & \mapsto 0 \\ c \oplus d & \mapsto \# \end{bmatrix} \\ b & \mapsto \begin{bmatrix} c & \mapsto 0 \\ d & \mapsto 1 \\ c \oplus d & \mapsto \# \end{bmatrix} \\ a \oplus b & \mapsto \begin{bmatrix} c & \mapsto \# \\ d & \mapsto \# \\ c \oplus d & \mapsto 1 \end{bmatrix} \end{bmatrix}$$

Let $\mathcal{Q}$ be the quantifier translating *two books*. Then the predicate *read two books* is not homogeneous: a read only one book, and b read only one book, so it is false of them, but a and b together, by closure under pointwise fusion, read two books.[16]

$$[\![\lambda x.\|\text{two books}\|(\lambda y.\|\text{read}\|(x,y))]\!] = \begin{bmatrix} a & \mapsto 0 \\ b & \mapsto 0 \\ a \oplus b & \mapsto 1 \end{bmatrix}$$

This is very unfortunate, as quantifiers are so far only defined for homogeneous functions and therefore not applicable to this predicate. But of course it is actually quite possible to have a quantifier in subject position here: the sentence (7) should, on the cumulative reading of *read*, simply be true in the assumed scenario.

(7) Two people read two books.

It will therefore be necessary to allow quantifiers to take non-homogeneous predicates as arguments, but this rather spoils the above treatment of quantifiers, which relied on them being defined only for homogeneous functions. This necessitates additional steps to ensure that quantifiers can take non-homogeneous functions as arguments, but still interact with trivalence in the right way. On this, see section 2.4.5.

2.4.2 *Non-Homogeneous Predicates*

Of course there are, as we saw in section 1.4, also non-homogeneous predicates such as *numerous* and *heavy*, which are, at first glance, underived. However, while denoted by a single word in natural language, these predicates, if one thinks about it, don't look like logically simple things. The usual logical form assigned to *numerous* is not a predicate constant, but a more complex λ-expressions:

16 Note the use of $\|\cdot\|$. Since what is under investigation is a logical language which is to be given a semantics that respects homogeneity, natural language expressions are not interpreted directly, but first translated into a logic language by the translation function $\|\cdot\|$. This is also the procedure in Montague 1974, but has since come somewhat out of fashion.

(8) $\|\text{numerous}\| = \lambda x.\mu(x) \geq s$, where s is some contextual standard.

This suggests the following: all predicate constants have homogeneous denotations, and non-homogeneous expressions in natural language are, in fact, logically complex. If measure functions are not constrained to be homogeneous, then that would immediately explain the correlation observed in section 1.4: that the core examples of non-homogeneous predicates involve measurement of some sort.

In order to apply homogeneity to a measure function, $\preceq$ needs to be defined on numbers. The obvious way to do this is identify it with the $\leq$ relation. Since $\leq$ is a linear order, all numbers overlap, and this trivialises the homogeneity requirement for measure functions. Thus, the source of non-homogeneity can actually be located in the relation $\leq$, which can be added to the logical vocabulary. The identity relation $\doteq$ should also be adapted to denote regular identity on the domain of numbers.

2.4.3 *Adverbial Quantifiers and Individual Parthood*

Even if predicates like *numerous* and *heavy* are taken care of in this way, we still cannot have adverbial quantifiers yet. Adverbial *all*, for example, maps a homogeneous predicate to a non-homogeneous predicate. Assume that there are two pluralities a and b. All members of a arrived, but no member of b did. Then the predicate *all arrived* is true of a and false of b. If this predicate were homogeneous, it would then have to be undefined of $a \oplus b$, but in fact it is just false.

(9) Context: *All the girls arrived, but none of the boys did.*

The girls all arrived.	TRUE
The boys all arrived.	FALSE
The children all arrived.	FALSE

This is something that cannot be stated just in terms of measure functions on individuals alone, but it can be stated with a combination of parthood and a measure function, in addition to an existential determiner.

DEFINITION 2.21. (Existential Determiner)

1. (Syntax) $\mathcal{E}$ is an expression of type `(et)t`.[17]
2. (Semantics) $[\![\mathcal{E}]\!]$ is that $\preceq_0$-minimal homogeneous function which is true of f and g iff there is an x such that $f(x) = g(x) = 1$.

If $\sqsubseteq$ is an object-language representation of $\preceq$, then adverbial *all* can be translated in the following way.

(10) $\|\text{all}\| = \lambda P.\lambda x.\mathcal{E}(\lambda y.y \sqsubseteq x \wedge \frac{\mu(x)}{\mu(y)} = 1)(P)$

Note that $\sqsubseteq$ cannot possibly be a homogeneous relation. More precisely, $\lambda x.\lambda y.y \sqsubseteq x$ cannot be homogeneous with respect to x: $a \sqsubseteq a \sqcup b$ (where $\sqcup$ represents $\oplus$) is a

17 Of course, this definition could also be generalised across types.

tautology, but $a \sqsubseteq b$ must still be able to be false.[18] However, if we allow a slightly altered version of $\preceq$ into the object language as a non-homogeneous logical constant, then it is possible to maintain that the only sources of non-homogeneity in the language are relations between numbers and the logical relation $\preceq$ between individuals.

DEFINITION 2.22. (Individual Parthood)

1. (Syntax) If α and β are expressions of the same type, then $\alpha \sqsubseteq \beta$ is an expression of type t.
2. (Semantics) $[\![\alpha \sqsubseteq \beta]\!] = \begin{cases} 1 & \text{iff } [\![\alpha]\!] \preceq [\![\beta]\!] \\ 0 & \text{iff not } [\![\alpha]\!] \between [\![\beta]\!] \\ \# & \text{otherwise} \end{cases}$

Note that there is non-homogeneity in two places, as it were. First, $\|$all$\|$ is non-homogeneous with respect to its argument x because $\sqsubseteq$ is non-homogeneous with respect to the argument position that it fills. And second, the restrictor of the existential quantifier is a non-homogeneous predicate.[19] The general strategy behind (10) it can be transferred to other quantifiers:

$$\|\text{two of}\| = \lambda x.\lambda P.\mathcal{E}(\lambda y.y \sqsubseteq x \wedge \mu(y) = 2)(P)$$

$$\|\text{more than half}\| = \lambda x.\lambda P.\mathcal{E}(\lambda y.y \sqsubseteq x \wedge \frac{\mu(x)}{\mu(y)} > 0.5)(P)$$

2.4.4 *Bare Plurals and Restrictors*

From the point of view of homogeneity, it is natural to think that the apparent gap between a sentence with an existential bare plural and its negation is also due to homogeneity: in a situation where Mary saw exactly one zebra, both (11a) and (11b) could just have the truth value #.

(11) a. Mary saw zebras. $\rightsquigarrow$ Mary saw more than one zebra.
b. Mary didn't see zebras. $\rightsquigarrow$ Mary didn't see a single zebra.

This idea is discussed from a more linguistic perspective in chapter 5. Here I would like to note that this follows from the formal system(s) discussed in this chapter if the plural noun *zebra* is true of pluralities of zebras and neither true nor false of atomic zebras. Assume that a and b are zebras and that Mary saw just one of them.

(12) a. $[\![\|\text{zebras}\|]\!] = \begin{bmatrix} a & \mapsto \# \\ b & \mapsto \# \\ a \oplus b & \mapsto 1 \end{bmatrix}$

18 Note that it is homogeneous with respect to the argument y. (i) is undefined if only some of the girls were among the performers.

(i) The girls were among the performers.

19 It is not a bivalent predicate, though; $\lambda y.y \sqsubseteq x \wedge \frac{\mu(x)}{\mu(y)} = 1$ is undefined of individual that is not x, but overlaps with x and has the same cardinality. This is of no consequence, however.

b. $[\![\|1 \text{ Mary saw } t_1\|]\!] = \begin{bmatrix} a & \mapsto 1 \\ b & \mapsto 0 \\ a \oplus b & \mapsto \# \end{bmatrix}$

The existential determiner from the previous section, applied to these two predicates, doesn't yield truth. But it doesn't yield falsity, either. Consider P, which is a $\preceq$-part of the meaning of the plural noun *zebras*.

$$P = \begin{bmatrix} a & \mapsto 1 \\ b & \mapsto 1 \\ a \oplus b & \mapsto 1 \end{bmatrix}$$

$\mathcal{E}$ is true of P and the property of being seen by Mary, because both are true of a. But since $P \prec [\![\|\text{zebras}\|]\!]$, homogeneity forces $\mathcal{E}$ to be undefined, rather than false, of the predicates in (12).

This relies on the plural noun, which functions as the restrictor, having a homogeneous denotation. It fits well with the idea that measurement is involved in non-homogeneity and that bare plurals are the only quantifiers that do not involve any kind of measurement of the restrictor. The pluralising operator that this requires is not definable in the object language, since it is not possible to define in it a homogeneous non-atomicity predicate.[20] But it is easy enough to add a pluralising and a singularising operator to the logical vocabulary of the language.

definition 2.23. (Number Operators I)

1. The meaning of the singularising operator σ is defined as that function which maps any unary predicate f to the $\preceq$-maximal predicate that agrees with P on the set $\{x \mid |x| = 1\}$.
2. The meaning of the pluralising operator π is defined as the function which maps any unary predicate P to the $\preceq$-maximal predicate that agrees with f on the set $\{x \mid |x| > 1\}$.

$[\![\sigma]\!](P)$ agrees with P on all atoms; and it is undefined of all pluralities. $[\![\pi]\!](P)$ does the exactly analogous thing, with atoms and pluralities switching roles: it agrees with P on all pluralities and is undefined of all atoms.

In section 1.5.4, I argued that homogeneity doesn't project from restrictors. If quantifiers were defined in such a way that they treated their restrictor arguments as if they were non-homogeneous, replacing all # with 0, then this theory of bare plurals couldn't work, since it relies on the undefinedness of the bare plural noun for atoms.[21] However, one could change the number operators so as to remove any undefinedness that is caused by homogeneous predication *within* the restrictor (such as a definite plural in a relative clause). If every noun phrase comes with

20 One can define a *non*-homogeneous non-atomicity predicate such as $\lambda x.\mu(x) > 1$ or $\lambda x.\mathcal{E}(\lambda z.z \doteq z)(\lambda y.y \sqsubseteq x \wedge y \neq x)$.

21 There is a version of George's theory (George 2008a) that does essentially that. Without going into the technical details, it predicts the following: homogeneity does not project from the restrictor if the restrictor is true of at least one individual. However, it also predicts that homogeneity doesn't project from the scope if the scope is true of at least one individual in the restrictor. This predicts

a number operator on top of it, this would take care of the non-projection from restrictors.[22]

For any homogeneous predicate that is not $\preceq_0$-minimal, there is a unique $\preceq_0$-minimal part which is true of the same individuals, but contains only the undefinedness that is absolutely required to fulfill homogeneity.[23]

FACT 2.9. *If f is homogeneous, then there is a unique $f' \preceq f$ such that f' is a $\preceq_0$-minimal homogeneous function.*

Instead of agreeing with their argument on the relevant set of individuals, the number operators now have to agree with the $\preceq_0$-minimal homogeneous part of their argument.

DEFINITION 2.24. (Number Operators II) Write f^0 for the $\preceq_0$-minimal homogeneous part of f.

1. The meaning of the singularising operator σ is defined as that function which maps any unary predicate f to the $\preceq$-maximal predicate that agrees with f^0 on the set $\{x \mid |x| = 1\}$.
2. The meaning of the pluralising operator π is defined as the function which maps any unary predicate f to the $\preceq$-maximal predicate that agrees with f^0 on the set $\{x \mid |x| > 1\}$.

Take, as an example, the predicate *student who read the books*. Assume that there are three students a, b, and c. a read all of the books, b read half of them, and c read none.

$$[\![\|\text{read the books}\|]\!] = \begin{bmatrix} a & \mapsto 1 \\ b & \mapsto \# \\ c & \mapsto 0 \\ a \oplus b & \mapsto \# \\ a \oplus c & \mapsto \# \\ b \oplus c & \mapsto \# \\ a \oplus b \oplus c & \mapsto \# \end{bmatrix}$$

Since all individuals are students, the above function is also the meaning of *student who read the books*. The $\preceq_0$-minimal homogeneous part of this predicate

that *all of* $a \oplus b$ should be false of P, which is in contradiction to the experimental findings for homogeneity.

$$P = \begin{bmatrix} a & \mapsto 1 \\ b & \mapsto \# \\ a \oplus b & \mapsto \# \end{bmatrix}$$

22 I am excluding mass terms from consideration at this point.

23 This follows from the fact that whether replacing # by 0 somewhere in f causes a homogeneity violation is independent of any other such replacements. It never happens that there are distinct x, y such that $f(x) = f(y) = \#$, and one can change the value of f to 0 for one of the two without violating homogeneity, but not for both.

is the function f below. In it, instances of # are replaced with 0 as long as this doesn't lead to homogeneity violations.

$$f = \begin{bmatrix} a & \mapsto 1 \\ b & \mapsto 0 \\ c & \mapsto 0 \\ a \oplus b & \mapsto \# \\ a \oplus c & \mapsto 0 \\ b \oplus c & \mapsto \# \\ a \oplus b \oplus c & \mapsto \# \end{bmatrix}$$

The number operators applied to $\|$read the books$\|$ thus agree with f on atoms and non-atoms, respectively, and are undefined everywhere else.

$$[\![\sigma(\|\text{student read the books}\|)]\!] = \begin{bmatrix} a & \mapsto 1 \\ b & \mapsto 0 \\ c & \mapsto 0 \\ a \oplus b & \mapsto \# \\ a \oplus c & \mapsto \# \\ b \oplus c & \mapsto \# \\ a \oplus b \oplus c & \mapsto \# \end{bmatrix}$$

$$[\![\pi(\|\text{student read the books}\|)]\!] = \begin{bmatrix} a & \mapsto \# \\ b & \mapsto \# \\ c & \mapsto \# \\ a \oplus b & \mapsto \# \\ a \oplus c & \mapsto 0 \\ b \oplus c & \mapsto \# \\ a \oplus b \oplus c & \mapsto \# \end{bmatrix}$$

As they are defined now, the number operators can take as arguments only homogeneous predicates (because non-homogeneous predicates have no $\preceq_0$-minimal homogeneous part). It is not clear to me that they need to be able to take non-homogeneous arguments for linguistic purposes, since non-homogeneous predicates in restrictors tend to be understood as appositives. However, replacing $\preceq_0$ with the relation $\preceq_0^H$, to be defined in section 2.4.5 below, takes care of this problem: the $\preceq_0^H$-minimal part of f is that function which is obtained by changing f so that it maps arguments to 0 instead of # without introducing any new homogeneity violations.

2.4.5 *The Homogeneity of Quantifiers*

In light of what has been discussed in the preceding sections, it is obviously necessary to admit non-homogeneous functions as denotations of expressions of the language, and to let quantifiers in general take them as arguments as well. But it is still possible to save a notion of homogeneity with which one can capture the behaviour of quantifiers in the face of non-homogeneous arguments. The crucial step is to use a relation that holds between f and g if f is a $\preceq$-part of g and f does not violate homogeneity any worse than g does.[24]

DEFINITION 2.25. For any function f, let $f^{\#/X}$ be that function which is just like f except that $f(x) = \#$ for all $x \in X$. Let $f|^H$ be $\{X \mid f^{\#/X}$ is homogeneous$\}$.

$f|^H$ is the set of all sets X such that if you change the value of f to # for all $x \in X$, then the result is a homogeneous function. Thus, $f|^H \subseteq \wp(\text{dom}(f))$. If f is homogeneous, then $f|^H = \wp(\text{dom}(f))$.

DEFINITION 2.26. (Homogeneity-Respecting Parthood)

1. For all individuals x, y, $x \preceq^H y$ iff $x \preceq y$.
2. $f \preceq^H g$ iff $f \preceq g$ and $f|^H = g|^H$.

DEFINITION 2.27. (Homogeneity-Respecting Overlap) $x \between^H y$ iff there is a z such that $z \preceq^H x$ and $z \preceq^H y$.

DEFINITION 2.28. (Homogeneity — new) A function f is *homogeneous* iff for all x, y, $x \between^H_{\text{dom}(f)} y \to f(x) \between^H_{\text{range}(f)} f(y)$.

These definitions may seem circular in that homogeneity is referred to within the definition of homogeneity-respecting parthood, which is in turn used in the definition of homogeneity (through overlap). However, everything is grounded in the identification of $\preceq^H$ with mereological parthood on the domain of individuals.

What we can now say is this: even though the domains of various types contain homogeneous and non-homogeneous functions alike, the range of the interpretation function $\mathcal{I}$ that interprets constant terms is still just the set of functions that are homogeneous in this sense (and also $\preceq_0$-minimal). Of course, if all domains contain only homogeneous functions anyway, then there is no change, but the crucial thing is that even if there are non-homogeneous functions in the domain, they don't count for the purposes of overlap and therefore don't disrupt our ability to quantify over pluralities. Consequently, all the results from section 2.2.2 apply. It is also perfectly possible to use $\preceq^H_{0|1}$ instead of $\preceq^H$ in order to obtain the desired behaviour for non-monotonic quantifiers.

2.4.6 *A Problem with Bound Pronouns*

Before closing this section, I would like to note a phenomenon that is fundamentally problematic for any approach to homogeneity that makes use of a compositional trivalent logic, which arises in combination with the traditional treatment of pronouns as bound variables. In particular, two predicates can be found which are assigned the very same semantic value, but which, when combined with *all the boys*, empirically show different homogeneity behaviour.

Take the sentence (13) and its two readings informally represented in (13a) and (13b). In (13a), the pronoun is bound by the distributivity operator in the matrix clause; in (13b), it is bound directly by the plurality.

24 This also allows the system to deal with predicates that are not homogeneous, but still undefined in some places.

(13) All the boys believe they will win their respective matches.

a. All the boys λx DIST λy believe y λu DIST λv will win v's match(es).
b. All the boys λx DIST λy believe x λu DIST λv will win v's match(es).

Intuitively, (13b) should be true if all boys believe that all boys will win, false if at least one boy believes that *no* boy will win, and undefined otherwise; i. e. it is homogeneous wrt the embedded pronoun, but not the matrix subject. It needs to be established that (13b) is actually a genuine bound reading and that we are not just dealing with simple coreference. The argument comes from two source. First, the predicate can be distributed over a conjunction of definite plural noun phrases.

(14) All the boys and all the girls believe they will win their matches.

(14) has a reading on which it is true if every boy believes that all the boys will win their matches and every girl believes that alle the girls will win theirs. Second, the pronoun can receive a sloppy reading under ellipsis.[25]

(15) All the boys believe they will win their matches, and all the girls do, too.

Unfortunately, the present theory makes weird predictions for the case of (15b). It is predicted to be false as soon as one individual doesn't believe that she *herself* will win. Assume for simplicity a universe with only two entities a and b, both of whom are boys. Further assume that a believes that both a and b will win, and b only believes that a will win.

$$[\![\text{1 believe they}_1 \text{ will win}]\!] = \begin{bmatrix} a & \mapsto 1 \\ b & \mapsto 0 \\ a \oplus b & \mapsto \# \end{bmatrix}$$

This function is the familiar predicate for which *all of* $a \oplus b$ is supposed to return 0. The problem is that, intuitively, for the purpose of homogeneity, we don't want to covary the pronoun with the argument. For homogeneity, the truth-value of "b believes that b will win" isn't relevant; what counts is the truth-value of "b believes that $a \oplus b$ will win".

Imagine further another scenario, in which both a and b believe that only a will win. Now it turns out that both readings, (13a) and (13b), are extensionally identical. And yet, *all the boys* ought to return 0 on the distributively bound

25 I thank Irene Heim (p. c.) for pointing out this argument.

reading (because here the pronoun legitimately covaries with the argument), and # on the collectively bound reading.[26]

$$[\![\|1 \text{ believe they}_1 \text{ will win}\|]\!] = \begin{bmatrix} a & \mapsto 1 \\ b & \mapsto 0 \\ a \oplus b & \mapsto \# \end{bmatrix}$$

$$[\![\|\textsc{dist } 1 \text{ believe they}_1 \text{ will win}\|]\!] = \begin{bmatrix} a & \mapsto 1 \\ b & \mapsto 0 \\ a \oplus b & \mapsto \# \end{bmatrix}$$

While on the one hand, this is a very deep and principled problem for the trivalent logic approach that I have pursued here, the standard static treatment of pronouns as bound variables is grossly simplistic, and I consider it likely that one may be hopeful that a proper analysis of pronouns will eventually take care of this problem.

2.5 CONCLUSION

In this chapter, I have proposed a particular approach to the formal treatment of the phenomenon of homogeneity and, in particular, its interaction with quantifiers. Building on work by Lepage (1992) on trivalent type theory, I have defined a logic with an algebraic semantics where the notion of parthood is generalised to all domains and all aspects of the behaviour of homogeneity flow essentially from a single constraint, which is stated in terms of the parthood relation and also applied across domains. As applied to predicates, this constraint yields homogeneity itself, as a condition on the positive and negative extension of a predicate. When it is applied to quantifiers, it constrains the projection behaviour of homogeneity in the right way, thus yielding a principled and fully general characterisation of natural language quantifiers with respect to homogeneity. Furthermore, the logic fulfills the intuitive desideratum that one should be able to uniquely derive the negative extension and extension gap of a predicate or quantifier from its positive extension.

A broader treatment of natural language semantics makes it necessary to allow non-homogeneous predicate expressions into the language. I have suggested, however, that they enter in only very specific places through certain additions to the logical vocabulary, which make it possible to define non-homogeneous predicates as λ-expressions. Predicate constants, however, are always required to be homogeneous by the definition of the semantics of the language. This fits well with an indirect interpretation procedure, where natural language is first

26 Note also that if *all* is inserted in the embedded clause, the result is another potentially non-homogeneous predicate to which quantifiers can, as a matter of fact, felicitously apply. Assume that a believes that a will win and b believes that b will win, that is, everybody believes only of themselves that they will win, so nobody believes that all of $a \oplus b$ will win.

$$[\![\|1 \text{ believe they}_1 \text{ will all win}\|]\!] = \begin{bmatrix} a & \mapsto 1 \\ b & \mapsto 1 \\ a \oplus b & \mapsto 0 \end{bmatrix}$$

translated into the logical language, which is then interpreted: non-homogeneous predicates like *numerous* are single words in natural language, but they are still logically complex and are not translated merely to as predicate, but as λ-expressions, which can have non-homogeneous denotations.

Two directions suggest themselves for immediate further development: first, the generalisation of this approach to a multi-sorted logic in which they facts discussed in section 1.3 can be fully captured. And second, the exploration of the extent to which a system like this might be useful for vagueness.

3 HOMOGENEITY AND NON-MAXIMALITY*

This chapter develops a theory of the exception tolerance of sentences with plural definite descriptions, which treats it as a pragmatic phenomenon that arises from the context-dependent interaction of the homogeneity of plural predication on the one hand with independent pragmatic principles on the other. This allows, among other things, an explanation for the dual effect of *all*: as a matter of its semantics, it removes homogeneity, but because that is one of the necessary ingredients for such exception tolerant readings, the function of *all* as a maximiser/"slack regulator" emerges as a consequence. This theory is then further explored in light of the picture of homogeneity that has emerged in chapter 1 and compared to previous approaches to the same phenomenon.

3.1 NON-MAXIMALITY: THE PHENOMENON

3.1.1 *Basic Observations*

It has long been known that the truth conditions of plural predications are not always strictly universal and we often judge such sentences as correctly describing a situation where there are, in fact, some exceptions. The precise extent of this tolerance seems to depend on various contextual factors. Following Brisson (1998), I call this phenomenon *non-maximality*. A classical example is (1) (from Lasersohn 1999), which one can easily see felicitously used to describe a situation in which there are, in fact, some townspeople who are still awake.

(1) The townspeople are asleep.

Besides this one, many more can be constructed. Imagine, for example, that we are trying to gauge the audience's reaction to Sue's talk, and (2) is uttered. This would seem quite appropriate even if the perpetually dour Prof. Smith, who is known to never smile anyway, is, in fact, looking neutral.

(2) The professors are smiling.

In some contexts, non-maximality can even go so far as to yield essentially existential readings. Malamud (2012) points out an example along the lines of (3). Here it seems that Mary's utterance simply means that enough windows are open to warrant going back to the house.

(3) Context: *Mary and John leave Mary's house to go on a road-trip. A few minutes into the ride, the following discourse takes place:*

* Sections 3.1 through 3.3, as well as section 3.6 contain material from Križ 2015, which is reproduced here with permission from Oxford University Press.

J: There is a thunderstorm coming. Is the house going to be okay?
M: Oh my, we have to go back — the windows are open!

Once one adds *all* to a plural sentence, however, this readiness to tolerate exceptions disappears (Brisson 1998, Lasersohn 1999). For example, if Prof. Smith didn't smile, even if it is known that he never does, it is strange to hear (4a), and very natural to react to it by uttering (4b).

(4) A: After her talk, Sue looked at the audience. All the professors were smiling.
B: No, Smith didn't. I know it doesn't mean much, but still.

3.1.2 *The Properties of Exceptions*

Despite these apparently not-quite-universal truth conditions of plural predications, it is not straightforwardly possible to mention the exceptions explicitly. It was already pointed out by Kroch (1974: 190f.) (cited in Lasersohn 1999) that sentences of the form of (5a) sound contradictory, while (5b) does not.

(5) a. #Although the professors are smiling, one of them is not.
b. Although more or less all the professors are smiling, one of them is not.

The same effect can be observed with *but* (*pace* Brisson 1998).

(6) a. #The professors are smiling, but one of them isn't.
b. More or less all the professors are smiling, but one of them isn't.

It is not entirely impossible to admit exceptions, but this can be done only in what feels like an aside that does not address the main point the speaker was making. This impression is strengthened by the obligatory presence of an adverbial like *of course*.

(7) a. The professors are smiling. Of course, not Smith, but you know, he never smiles, it doesn't mean anything.
b. The townspeople are asleep. Of course, the gatekeeper is probably still up, but we know that's he's always there anyway.

The intuitive conclusion I draw from this, and which has been drawn by Lasersohn (1999), is that plural sentences allow only for exceptions that are irrelevant for the current purposes of the conversation: the sentence can be used as long as we are, for current purposes, close enough to its being true on a strict universal reading. Lasersohn also provides a scenario in which every exception would matter and in which a plural sentence consequently cannot be interpreted non-maximally:

> "Suppose we are conducting an experiment on the nature of sleep. We have several people serving as experimental subjects there in our lab, lying on beds, dozing off one by one. In order for the experiment to proceed, we need to make sure that all of them are completely

asleep; otherwise the experiment is ruined. In this sort of situation, if you assert (9), every last one of the subjects had better be asleep; exceptions are not tolerated.

(9) The subjects are asleep."

It is worth pointing out another aspect of this exception tolerance that has not hitherto received explicit attention in the literature: it is not only the number and identity of the exceptions that influences whether they are acceptable, but it also makes a difference what they do instead of fulfilling the predicate.

To illustrate, take up again the example of the smiling professors. We will readily tolerate a non-smiling Smith and still judge "The professors smiled" true if Smith just had a neutral expression on his face, especially if he is known to smile only rarely. If, however, he looked visibly angry, the judgment seems to change and we are less prepared to still accept the sentence as true. More likely, it would count as neither true nor false in such a situation Similarly, when a small number of townspeople are having a noticeable party in the street, we would be much less inclined to call the sentence "The townspeople are asleep" true than in a situation where the very same people are at home and reading quietly.

3.1.3 *Non-Maximality and Negation*

It is worth noting that one finds analogous behaviour also for negated sentences. Recall that by homogeneity, (8) is true only if (almost) none of the students knew how to solve the problem. Non-maximality again permits some slight deviation from strict universality. If a professor, after grading an exam, utters (8), this will be an appropriate description of a situation where only one or two students knew what to do. The point she wants to make is presumably that this was a really bad class, or maybe she is admitting that she didn't do a good job of teaching them, and in that context, the few exceptionally smart individuals don't detract from her point.

(8) The students didn't know how to solve the problem. (Of course, Alice and Bob got everything right, but they are really exceptionally smart and you can't compare them with the others.)

In contrast, if she were to say (9), *none* being the negative analogue of *all*, even Alice and Bob would falsify her utterance and she would not be giving an appropriate description of the situation.

(9) None of the students knew how to solve the problems

3.1.4 *The Connection with Homogeneity*

In the preceding two chapters, I have discussed *all* as a homogeneity remover. But the word has perhaps received more attention in its capacity as a *slack regulator* (e. g. Lasersohn 1999, Brisson 1998): it eliminates the possibility of non-maximal uses. For example, if Prof. Smith didn't smile, even if we know that he never does, it is impossible to accept (10a).

(10) All the professors were smiling.

If a theory could be found that predicts non-maximality to arise only if there is homogeneity, then the pragmatic effect of *all* as a slack regulator would elegantly follow from its purely semantic effect of removing homogeneity. Note that this happens not only with definite plurals: predications that are homogeneous with respect to other kinds of parthood than individual parthood also allow non-maximal readings, and it just so happens that the homogeneity removers associated with those domains also block non-maximality (cf. section 1.3.1).

(11) a. The forest is dense.
b. The book is intelligently written.

(11a) is quite compatible with a few sparser patches as long as they don't significantly diminish the potential for getting lost in the forest, and in most contexts, one can surely say that a book is intelligently written even if a chapter or so falls short of the highest standards. But once the requisite homogeneity removers are added, such exceptions are no longer permitted.

(12) a. The forest is dense everywhere.
b. The book is intelligently written throughout.

In chapter 7, I will further discuss how a number of other constructions that were identified as showing homogeneity in section 1.3 can be viewed in terms of non-maximality as well: conditionals and generics are, of course, well-known for permitting exceptions, though these have generally been analysed quite differently. Furthermore, I will argue that embedded questions can be understood non-maximally as well and suggest that this is, in fact, what is behind *mention some*-readings.

3.1.5 *Non-Maximality Isn't Reference Restriction*

I would like to establish right at the outset one way in which this phenomenon cannot be explained: non-maximal readings do not arise through salience- or relevance-based domain restriction of the definite description.

Individuals that are outside of the domain are not included in the reference of a definite description and are simply not being talked about. Bringing them up as exceptions is a *non sequitur*. This is exemplified in the discourse in (13). However, exceptions that were ignored by way of non-maximality can always be brought up by an interlocutor, prompting the original speaker to justify glossing over them, as shown in (14).

(13) *Uttered at the ENS in Paris.*
A: The students are happy.
B: #Well, actually, the students at the Sorbonne aren't.

A′: What? I wasn't talking about THEM.

(14) A: The professors smiled.
B: Well, actually, Smith didn't.
B′: Well, yeah, but you know, he NEVER does.

That the reference of a definite plural is always maximal even when the predication is understood non-maximally can be seen in cases of predicate conjunction and when anaphoric pronouns are involved. With predicate conjunctions, it is perfectly possible to have a reading that is non-maximal with respect to one conjunct, but maximal with respect to the second conjunct. This can be enforced by adding adverbial *all* in the second conjunct without detracting from the possibility of non-maximality in the first, as shown by (15b). However, it is clear that the second conjunct is maximal with respect to the whole group, including the exceptions ignored by non-maximality in the first conjunct, which causes (15b) to be impossible.

(15) a. Context: *All the professors except Smith smiled, and then all the professors, including Smith, left.*
The professors smiled and then all left the room.
b. Context: *All the professors except Smith smiled and then left, leaving Smith behind.*
#The professors smiled and then all left the room.

Similarly, *they* in (16) doesn't automatically refer to just those professors who smiled.

(16) The professors smiled. Then *they* (all) stood up and left the room.

Salience-based reference restriction patterns differently from non-maximality with respect to the above. The example in (17) is inspired by Schlenker 2004. It seems clear that *they* only refers to the girls who have to go to the bathroom.

(17) Context: *A group of ten boys and ten girls are on an excursion with their teacher B. Three of the girls raise their hands to indicate that they need to go to the bathroom.*
A: Wait, the girls need to go to the bathroom.
B: Okay, but *they* will have to catch up with the rest of us.

Imagine further one of the girls who have *not* raised their hand bringing up herself as a supposed exception. This is likely to be perceived as a *non sequitur*, further setting apart this case of actual restricted reference from non-maximality.

(18) A: Wait, the girls need to go to the bathroom.
G: #Well, actually, some of us don't. . .

3.1.6 *Interim Summary*

Sentences with definite plurals can often be used to describe situations where not absolutely all members of the plurality in question fulfill the predicate, a phenomenon known as non-maximality. I have pointed out, following Lasersohn 1999, that this is only possible to the extent that these exceptions are somehow contextually irrelevant and consequently cannot easily be explicitly mentioned. Furthermore, I have argued for a link between homogeneity and non-maximality, based on the observation that only sentences which have an extension gap due to homogeneity allow for non-maximal readings. Once an element is added that removes homogeneity, such as *all*, the potential for non-maximal readings disappears as well. Finally, I have made the case for distinguishing non-maximality from any sort of salience-based reference restrictions, as it can be shown that even in sentences that are understood non-maximally, the reference of the definite description is still the maximal plurality of the relevant sort and does not exclude the exceptions.

3.2 A THEORY

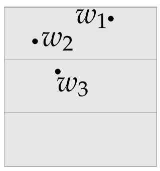
In this section, I present a theory on which non-maximal readings can be derived as conversational implicatures, in particular as *quality implicatures*. Starting with an exposition of pragmatic background assumptions, I will spell out formally how such implicatures are derived. I then further propose a principle that prevents them from arising for sentences that do not display the homogeneity property, such as those with *all*.

3.2.1 *The Current Issue*

In section 3.1.2, I stated the intuition that non-maximal readings allow for exceptions as long as the exceptions are somehow irrelevant for current purposes. This means that we need to operationalise the notion of *current purposes* to use it in a formal theory. They are represented by a partition of the set of possible worlds, which I will call an *issue*. I assume that speakers always posit such an issue that the conversation aims at resolving, where to resolve the issue it to determine which of its cells contains the actual world. The idea of interpreting definite plurals against the backdrop of such a partition was first proposed by Malamud 2012, inspired by van Rooij's (2003) use of such in the interpretation of questions. I will say more about what exactly I take to be the nature of this issue in section 3.3.6, when the mechanics of the theory are in place.

For concreteness, let me establish the following extremely simplistic scenario for future use: we are interested in how Sue's talk was received, and right now we are only going to judge it based on the facial expressions of the professors in the audience. We partition the set of possible worlds into three cells: a cell i_1 where Sue's talk counts as well-received, a cell i_2 where the reception is mixed, and finally i_3, where it was ill-received.[1]

1 Presumably, there is some vagueness or uncertainty as to where the borders of these cells are. This is a separate issue that is of no concern here.



i_1: positive reception
i_2: mixed reception
i_3: negative reception

A world where all the professors smiled, say, w_1, is obviously in i_1. Let w_2 be a world where all the professors smiled except Smith, who looked neutral, as he almost always does. Such a world will also be in the cell for *positive reception*. If, however, only half of the professors smiled (w_3), we'll count this as a mixed reception.

3.2.2 *Quality Implicatures*

Given this formalisation, we can now approach a definition of what it means for exceptions to be irrelevant for current purposes: their presence doesn't influence which cell we are in; we are still in the same cell that we would be if there were no exceptions. A sentence that is true modulo such irrelevant exceptions will be called *true enough*.[2]

(19) sufficient truth
We write $\approx_I$ for the equivalence relation that holds of two worlds u, v iff u and v are in the same cell of I. A sentence S is *true enough* in world w with respect to an issue I iff there is some world w' such that $w' \in [\![S]\!]^+$ (S is literally true in w') and $w \approx_I w'$.

In terms of the example above, the sentence "The professors smiled" is not true in w_2, but it is true enough, because w_1, where it is literally true, is in the same cell.

The next ingredient of our theory is a change in the maxim of quality. As one of the Gricean maxims of conversation (Grice 1975), it is traditionally stated as the imperative to make only true statements (to the best of one's epistemic ability). But by definition, the purpose of the conversation is only to resolve a certain issue — to learn which cell of the issue the actual world is in — , so this is an unnecessarily strong requirement. We therefore suggest that the maxim of quality is in fact weaker and requires only that one should say sentences that are *true enough* for current purposes.

(20) (weak) maxim of quality
A speaker may say only sentences which, as far as she knows, are true enough.

This might seem radical, but when the whole theory is in place, we will see that speakers are still not allowed to say something that is actually *false*, so that its effect is restricted to sentences with an extension gap.

The weakened maxim of quality gives rise to systematic quality implicatures: the information that is communicated by a sentence is not its literal truth-

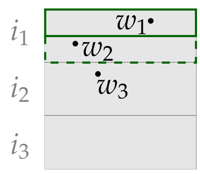
2 We thank Roger Schwarzschild for suggesting this manner of presentation.

conditions, but rather the union of all question cells that are compatible with (i. e. have a non-empty intersection with) its positive extension. For an example, take the interpretation of (21) in light of our toy issue.

(21) The professors smiled.

Even if the speaker knows that the sentence is not literally true and that we are in w_2 rather than w_1, the maxim of quality still permits her to utter the sentence. Knowing this, a hearer can infer no more than that we must be in i_1, and so the proposition[3] communicated by (21), given the current issue, is simply i_1. More generally, the procedure for arriving at the communicated meaning is to simply extend the literal meaning of the sentence to the closest cell boundaries. This is reflected by the dashed line below.

$[\![\textit{The professors smiled.}]\!]^+$



w_1: all smiled

w_2: Smith neutral, rest smiled

w_3: only half smiled

Note that at the same time, we predict that a plural is interpreted maximally when it is in fact the case that every single exception would be relevant, as in Lasersohn's sleep study scenario (cf. section 3.1.2).

3.2.3 *Addressing an Issue*

So far, nothing prevents us from applying the same reasoning to a sentence without an extension gap like (22).

(22) All the professors smiled.

After all, an utterance of this sentence in w_2 still complies with the maxim of quality: we are in a world that is in the same cell as one where it is literally true (e. g. w_1). However, (22) is just *false* in w_2 and cannot be appropriately used to describe it. More generally, *any* sentence cannot be used when it is literally false, only when it is either true, or neither true nor false.

We suggest that what is behind this is a restriction on which sentences can be used to address an issue: a certain alignment is required between the two.

(23) ADDRESSING AN ISSUE
A sentence S may be used to address an issue I only if there is no cell $i \in I$ such that i overlaps with both the positive and the negative extension of S, i. e. S is true in some worlds in i and false in others.

This condition may be seen as a way of extending Lewis's (1988) notion of *aboutness* to sentences with extension gaps, where aboutness is defined as follows:

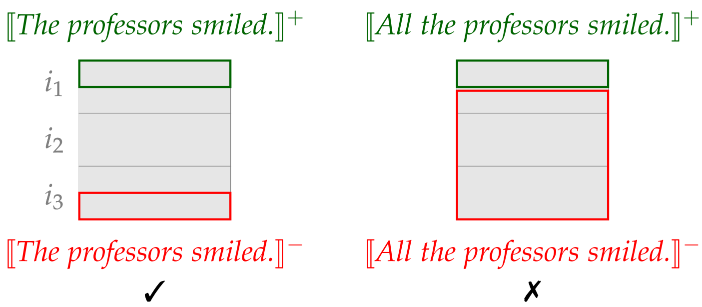
3 Here and always, we use the term *proposition* in its technical sense to mean a set of worlds.

(24) S is *about* I iff $\forall i \in I : i \subseteq [\![S]\!] \vee i \cap [\![S]\!] = \varnothing$.

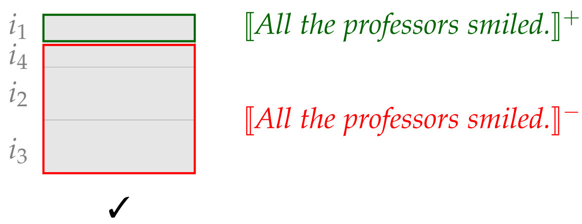
This means that the worlds in any given question cell may not fall on different sides of the true-false boundary of the sentence. It is this formulation which we have generalised to three-valued sentences: the worlds in one cell may not fall on different sides of the boundary, but, now that the boundary is extended, they may fall *onto* it. The worlds that fall onto the boundary (i. e. into the extension gap) somehow don't count; they are hushed up, which fits intuitively with the fact that the extension gap of a plural sentence is, in a way, not to be spoken of.

Another way of regarding the condition in (23) is an extension of the principle of non-contradiction to the level of communicated content: just as a sentence cannot have a positive extension that overlaps with its negative extension, a sentence cannot be used in a context where the communicated meaning of its positive version would overlap with the communicated meaning of its negative counterpart.[4]

As applied to our example, condition (23) entails that the *all*-sentence (22) simply cannot be used to address the issue, because its positive and negative extension are both compatible with i_1.



It could only be used if the issue were different, i. e. if we cared whether really all professors, even Smith, smiled. This is a perfectly sensible issue, too: a speaker may find it worth pointing out that even Smith, who almost never smiles, did smile. By using (22), she can convey this information.



Thus, given that the *all*-sentence was, in fact, used, the speaker must take herself to be addressing an issue where every exception would matter, and so

4 Note, though, that this line of thinking is at odds with the paraconsistent view on vagueness. People have been reported to agree to (i) in a situation where John is a borderline case of tallness, cf. Sauerland 2011 and Ripley 2011. Cf. also section 1.7.2.

(i) John is both tall and not tall.

she must intend a maximal meaning. Hence, no weakening quality implicature is available.[5]

More generally, it follows that no sentence can be used when it is literally false. For assume that the actual world w is in the question cell i_1, and S is false in w. Then either S is not true in any world in i_1 and therefore eliminates i_1 as a possible answer to the current issue, in which case it is obviously inappropriate because the right answer shouldn't be eliminated. Or alternatively, S is true in some of the worlds in i_1, but then it is false in others in the same cell (including w). This means, by (23), that S cannot be used to address the issue at hand.

An immediate consequence of this is that sentences without the homogeneity property cannot be used imprecisely: the only way for such a sentence not to be literally true is to be false, and we just saw that a sentence cannot be used when it is false; so a sentence without an extension gap can only be used when it is literally true.[6]

3.2.4 *Interim Summary*

The upshot of what has been said is, in general terms, this: a sentence S can be used to describe a situation w iff (i) S is not false in w, and (ii) w is, for current purposes, equivalent to some situation in which S is literally true. Since non-homogeneous sentences are [not false] only when they are true, it follows that they can only be used when literally true.

This is a consequence of the interaction of two components of the theory. The first component is a weakened maxim of quality, which causes a sentence's communicated meaning to be the set of worlds which are, for current purposes, equivalent to a world in its literal positive extension. The second component is a condition on which issues a sentence can be used to address, requiring a certain kind of alignment between the sentence's meaning and the distinctions that are at issue: a sentence can not be used if some worlds in its positive extension are, for current purposes, equivalent to some worlds in its negative extension.

With this theory in place, I will now proceed to explore a number of further consequences and applications of it.

3.3 Some Consequences

3.3.1 *The Unmentionability of Exceptions*

We are now in a position to explain the properties of exceptions that we noted in section 3.1.2. Recall that it is not possible to mention exceptions explicitly

5 This is similar in spirit to what Lauer (2012) says of *exactly*: he assumes that the function of *exactly* is to mark that the speaker takes tiny differences with respect to a quantity to be relevant.

6 As pointed out by a reviewer, imprecise uses of numerals and descriptions of location are *prima facie* counterexamples to this. Our approach does seem to be incompatible with, though not entirely dissimilar in spirit from, Lauer's (2012), but it is not at odds with what Krifka (2002, 2007) suggests. In fact, Krifka's theory can even be translated surprisingly faithfully into our framework. See appendix 3.A of this chapter for such a translation.

without further ado, as evidenced by the fact that the sentences in (25) are always infelicitous.

(25) a. #Although the professors smiled, one of them didn't.
b. #The professors smiled, but/while/and one of them didn't.

This was interpreted, following hints by Lasersohn (1999), as indicating that exceptions have to be in some sense irrelevant in order to be permitted — an idea that is now implemented in our formal theory.

If the current issue is such that the plural statement may be used non-maximally, then the proposition that mentions the exception cannot be relevant to it. This is so because in order for the plural to be interpreted non-maximally, there must be a cell (call it i_1) in the current issue that contains both exceptionless worlds (u among them) and worlds with exceptions (call one of them v). But the exception-mentioning sentence E is false in the exceptionless world u; thus, the cell i_1 contains both a world where E is true (namely v) and one where it is false (viz. u), and so, by (23), E cannot be used to address this issue.

It can also be seen that if certain adverbials, in particular *of course*, are employed, it is possible to mention exceptions after all. I have nothing profound to say about this, but would like to note that it seems to me that the function of *of course* is to somehow signal that a shift to a more fine-grained issue is to be performed which is necessary to make relevant the utterance that is to follow. A deeper investigation of this and other adverbials (*actually*, *indeed*, and *in fact* would seem to be obvious candidates to look at) will have to be left to future research.

3.3.2 *What Exceptions Do*

An important prediction which sets the present theory apart from previous approaches to the same phenomenon is the following: for determining whether an individual is tolerated as an exception to a plural predication in a given situation, it matters not only who that individual is, but also what they do instead of fulfilling the predicate. We will readily tolerate a non-smiling Smith and still judge *The professors smiled* true if Smith just had a neutral expression on his face, especially if he is known to smile only rarely. If, however, he looked visibly angry, the judgment seems to change and we are less prepared to still accept the sentence as true. More likely, it would count as neither true nor false in such a situation.

This issue has not been recognised in previous treatments of non-maximality, which, as will become clear in section 3.6, were formulated in terms of a comparison between different individuals — for example, all the professors together, and the professors without Smith. This makes it difficult to bring the particulars of the deviant individuals' behavior into the picture, although it is not altogether impossible, as I will demonstrate for the case of Malamud 2012 later in section 3.6.3.

3.3.3 *Collective Predication, Team Credit, and Non-Maximality*

It is well known that collective predicates do not always require that every single part of a plural subject really participated in the action that is ascribed to the plurality.[7] (26), for example, is easily judged true of a situation where some of the boys actually just sat on the shore and watched the others without contributing to the effort (example from Brisson 1998).

(26) The boys built a raft.

One might think that being a bystander in this particular way semantically simply counts as participating in the collective action here, which, of course, raises the question of what the exact conditions are under which this happens. Brisson, however, already suggests that what one sees here is just another instance of non-maximality, and the theory I have presented offers a more fleshed-out perspective on this: being a bystander who does nothing of note is not to participate, but it is a way of being an irrelevant exception for the purposes of non-maximality.

In the context where a sentence like (26) would usually receive a team credit reading, say, when what is being discussed is the activities that the boys engaged in at a summer camp, it can easily been regarded as irrelevant whether really all of the boys participated in the building of the raft, as long as those who didn't were around in the same area and did nothing else of note. The theory furthermore predicts that if they did do something of note, then (26) is not appropriate to describe the situation; for example, when they wandered off into the woods instead.

More generally, I submit that the reason why collective predicates frequently tolerate a rather large number of exceptions lies in the kind of issues that tend to arise with them: when the verb is a subject-collective transitive predicate, then the point of the statement is usually to convey that the result was brought about — that a raft was created, for example. This naturally takes the focus off the details of participation; what tends to matter is only that the subject plurality was somehow around and none of its members did anything else of particular interest. Once it is explicitly relevant whether all of the individuals participated, for example, because those who did receive particular rewards, then the possibility for non-maximal readings seems to disappear.

For those speakers who accept *all* with such predicates, it is furthermore predicted that its addition enforces universal participation.[8]

(27) All the boys built a raft. / A raft was built by all the boys.

Since non-maximality is linked to homogeneity, we should also expect loose readings that are based on upward and sideways homogeneity. This sets the present theory apart from all other approaches, which have not paid any attention

7 The *locus classicus* for this observation is Dowty 1987. The phenomenon is frequently referred to as *team credit* in the literature after Lasersohn 1990: 194.

8 It may be difficult to obtain a collective reading here for the simple reason that it is hard to imagine why universal participation should matter; but the use of *all* presupposes that it does, via the Addressing constraint.

to the question of whether one can ignore not only exceptions, but also, in some cases, additional participants. While not ubiquitous, plausible examples of such exist. Small children frequently do not perform complicated manual tasks unaided, and so (28) could easily be considered true even when some of the actual glueing was done by a parent.

(28) The boys built a model plane.

In light of sideways homogeneity, it should also be possible to ignore both irrelevant exceptions from the original group and additional agents at the same time, which, given that both are possible separately, would not be surprising. And indeed, it seems to me that if a group of boys built a raft assisted by an adult, with some of them not actually participating, but being around and not doing anything of note, one may well describe this by saying that "the boys built a raft".

3.3.4 *Conjunctions*

If conjunctions denote individual sums, and consequently participate in homogeneity phenomena (cf. section 1.3.7), then the present theory would predict that they can also partake in non-maximality, which they generally don't. However, if team credit is considered to be an instance of non-maximality, then examples may be found.

(29) *John planted a tree. His little daughter Mary stood by him and watched intently.*
Mary: We planted a tree!
Susan (mother): John and Mary planted a tree. Well, of course, John did the work and Mary actually just watched.

The fact that Mary's mother would use *of course* and *actually* to accompany the mention of the exceptions is just what one would expect from non-maximality.

Still, there is no doubt that, if they are possible at all, non-maximal readings for conjunctions are very rare and hard to obtain. We can merely offer an intuitive consideration to make sense of this: If an individual weren't relevant to the current issue, it wouldn't be listed explicitly in a conjunction. Its mention will thus prompt the hearer to accommodate a current issue relative to which no non-maximal reading of the conjunction is possible.[9]

3.3.5 *A Puzzle: Numerals in Definite Descriptions*

Plural definite descriptions containing numerals, such as in (30), pose a puzzle for any theory of non-maximality: their reference is the same as that of a description without the numeral, uttered in the same context, but non-maximal readings seem to be much more difficult, perhaps impossible, to obtain with them.[10]

9 A similar view is expressed by Schwarzschild (1996: 92).
10 I thank Benjamin Spector and an anonymous reviewer of the Journal of Semantics for bringing this to my attention.

(30) The ten professors smiled.

The theory I have presented does not predict this behaviour, and I can only offer some speculations on what might be going on. Perhaps the overt mention of the numeral indicates that the speaker takes the number of professors to be relevant, and this makes it likely that she also takes the precise number of professors who smiled to be relevant. As hearers, we therefore make corresponding assumptions about the current issue, so that this number makes a difference. Then one world where only nine professors smiled is not equivalent to one where all ten did.

This presupposes that the addition of a numeral inside the definite description doesn't interfere with homogeneity. If it does, then of course there is no problem, as non-homogeneous sentences are not expected to show non-maximality. While I am intuitively inclined to believe that definite descriptions with numerals are homogeneous in English, it has been pointed out to me that they may not be in French. In particular, in a negated sentence with such a definite description, it is possible to explicitly state that some individuals do fulfill the predicate, as in (31a). This is not possible when the numeral is absent, in which case a homogeneous reading is obtained and exceptions are unmentionable as usual, shown in (31b).

(31) Jean devait rencontrer trois étudiants pour leur parler de son projet.
John had to meet three students to talk with them about his project.

a. Il arriva à l'heure au rendez-vous, mais il ne parla pas avec les trois étudiants. Il parla seulement avec l'un d'entre eux.
He arrived on time, but he didn't speak with the three students. He only talked to one of them.

b. Il arriva à l'heure au rendez-vous, mais il ne parla pas avec les étudiants. #Il parla seulement avec l'un d'entre eux.

As a potential locus of cross-linguistic variation, this matter is deserving of further investigation in the future.

3.3.6 *More on the Current Issue*

The idea that every conversation aims at resolving one or more questions is widely employed in the literature on information structure (cf. the seminal Roberts 1996). The idea is that the participants of the conversation keep a stack of questions, the *question under discussion* (QUD) stack. The top element of this stack is often called the QUD *simpliciter*. Simplifying greatly, a speaker has essentially two options at every point: either give an answer to the QUD, which, once fully answered, will be removed from the stack; or put a new question on the stack by asking it overtly.

It would of course be desirable to identify the current issue, as postulated in the present theory, with this QUD. This makes highly testable predictions, as one can just overtly ask a question, thereby setting the current issue, and then see how a definite plural in an answer is interpreted. Unfortunately, the predictions that the theory then makes are incorrect: if non-maximal readings were calculated based on the immediate last question raised, then weaker readings are predicted than we actually find.[11]

(32) Context: *In order to pass, Peter has to solve either all the math problems, or at least half of them and write an essay on mathematical Platonism.*

   a. Did Peter pass the exam?
   b. Yes, he solved the math problems.
   c. Yes, he passed the exam.

(32b), to the extent that it is felicitous as an answer here, is understood as saying that solving all the math problems was *how* Peter passed. But our theory cannot explain why it can only be understood this way: it should at least *also* have a reading where it adds no extra information and means just the same thing as (32c). The question is only whether Peter passed. All the worlds where *He solved the math problems* is literally true are worlds where he passed, and all the worlds where it is false are worlds where he didn't pass (since passing requires, in any case, at least solving some of the problems). Thus, Addressing is satisfied. Non-maximality would then weaken the meaning so that it is equivalent to saying just that Peter passed.

This wrong prediction seems to arise because our theory doesn't constrain the information conveyed by the quality implicature to actually be only about the math problems.[12] Somehow a sentence about math problems also manages to convey information about an essay on mathematical Platonism, in a way that is intuitively strange. It is precisely the same feature of the theory — that it compares worlds for equivalence, and not individuals — that allows us to take into account the properties of exceptions and causes this problem.

But things are actually worse: the same problem surfaces also with questions that are only about the plurality in question. For example, if A's question alone were the current issue, then B's answer in (33) should just mean that some of the professors smiled. A's question partitions the set of possible worlds into just two cells, c_1 where no professors smiled, and c_2 where at least one professor smiled. c_1 is just identical to the negative extension of *The professors smiled*, while the positive extension of that sentence is a proper subset of c_2. The Addressing constraint is therefore fulfilled. Furthermore, non-maximality widens the meaning to the nearest cell boundaries, so the communicated meaning of the sentence should just be c_2.

(33) A: Did any of the professors smile?
   B: Yes, the professors smiled.

And yet B's answer does seem to say something as weak as that. It doesn't necessarily mean that all of the professors smiled, but one does get a distinct impression that it wasn't just a single one. This means that in the above examples,

11 This point was brought up by Benjamin Spector (p. c.) and an anonymous reviewer of the Journal of Semantics.

12 Note that it isn't even clear what *being about the math problems* actually means in precise terms.

the question that was immediately asked cannot be identified with the current issue that is used for the purpose of interpreting the definite plural.

Recall that the starting point was the intuitive notion of *equivalence for current purposes*. What speakers of English mean when they use the phrase *current purposes* is rarely just the immediate last question that has been asked in the conversation. Rather, it would seem that they refer to something like the overarching goals of the participants, as relevant to this conversation. This is what we take the current issue to represent. It is, of course, strongly underdetermined by linguistic material and so subject to massive uncertainty and probably also vagueness. Questions asked and assertions made by a speaker do convey some information about what they take to be the purposes of the conversation, since they are constrained by considerations of relevance and the condition for Addressing an Issue. However, it is not possible to just ask a single overt question and pretend that the meaning of this question just *is* the current issue, not even as a hypothesis in a thought experiment. Presumably, the part of our brain that is responsible for reconstruction intentions and accommodating a context is autonomous enough so that it cannot be directly manipulated by a linguist's intentions.

This weakens the predictive power of the theory, since one cannot set up a context so precisely as to fully constrain the current issue and put a prediction to the test. The current issue is not directly accessible and cannot be so easily manipulated. One is therefore forced to restrict oneself to considerations of plausibility. It seems to me that the predecessors of our approach, van Rooij 2003 and Malamud 2012, intend their use of decision problems (and the partitions derived from those) to be understood in the same spirit. Indeed, I believe that no satisfactory of pragmatics can ultimately do without it, so that this is no undue proliferation of theoretical notions. It is possible that even quantity implicatures are, in fact, computed not with respect to the top element of the QUD stack, but with respect to the current issue as we understand it. Otherwise, it would be predicted that (34b), in reply to (34a),[13] could never implicate that Mary didn't eat all of the apples. However, it seems to us that this implicature is, in fact, present to the extent that the asker of (34a) can be assumed to care about distinctions that are finer than those made by her simple existence question.

(34) a. Did Mary eat some of the apples?
b. Yes, she ate some of them.

Similar points have been made by Geurts (2010). Thus, it is very plausible that there is such a thing as the current issue, and given this, the present theory is still not without merit: it provides a mechanism that explains the correlation between extension gaps and non-maximal readings, and it allows one, if not to make strict predictions, at least to make sense of observed data after the fact: in (32), it is quite likely that the manner of Peter's passing the exam is relevant for current purposes; at least B cannot be certain that it is not relevant, and so it would be hazardous of her to attempt to use the definite plural non-maximally. The answer (32b). Similarly, in (33), the question could easily be a prelude to asking who,

13 No focal stress on *some* is intended in either sentence.

exactly, it was that smiled, or how many of them smiled, so assuming that the overtly asked question is all that matters would be rather bold. Consequently, B would not attempt to use the definite plural just existentially.

3.3.7 *Homogeneity as a Presupposition*

I have argued in section 1.7.1 that homogeneity is probably not a presupposition, and this is required by the theory of non-maximality that I have presented. In worlds where a sentence's presupposition is not fulfilled, it is neither true nor false, and so presuppositions give rise to a kind of extension gap as well. The present theory could then be applied to these extension gaps as well and would make the following prediction: a sentence S with a presupposition p can be used despite p being false if and only if the actual situation is, relative to the current issue, equivalent to one where both p and S are true. The following is a potential candidate for such behaviour.[14] It is well-known that a singular definite description can sometimes be used in apparent violation of the uniqueness presupposition. The sentences in (35) are typical examples. It is conceivable that this is possible precisely because in the contexts where such usage occurs, the fact that the uniqueness presupposition is violated is irrelevant and so the actual situation is equivalent to one in which there is only one entity of the relevant kind.

(35) a. John took the elevator.
b. Mary met her sister yesterday.

This, however, cannot be correct, as following wrong prediction shows. (36) a suffers a presupposition failure if not all of the professors smiled. Assuming that an extension gap due to a presupposition failure is the same kind of thing as a homogeneity violation, this leads to the following trivalent meaning.

(36) Bill knows that all the professors smiled.
true if all the professors smiled and Bill knows it.
false if all the professors smiled and Bill doesn't know it.
neither if not all the professors smiled.

So in a situation where all the professors except Smith smiled and Bill knows that, (36) is neither true nor false. But as long as it doesn't matter for current purposes, our theory predicts that the sentence should be usable in this situation with a non-maximal reading. This is clearly not correct. Such a non-maximal reading is only available for (37), which does not contain the homogeneity-remover *all* in the embedded clause.

(37) Bill knows that the professors smiled.

To explain this difference, it is necessary to retain a more fine-grained view of the meaning of (37) which keeps apart homogeneity violations and presupposition failures.

14 This was suggested by Philippe Schlenker (p. c.).

(38) Bill knows that the professors smiled.

pres **true** if all the professors smiled.
false if none of the professors smiled.
neither if some but not all of the professors smiled.
ass **true** if in all of Bill's belief worlds, all the professors smiled.
false if in at least one of Bill's belief worlds, no professor smiled.
neither otherwise.

Non-maximality appears not only with respect to the assertion, but also with respect to the presupposition; that is to say, a sentence can sometimes be used when its presupposition is undefined. More generally, it seems to us that this is how non-maximality interacts with presuppositions: a sentence can never be used when the presupposition is false, but when the presupposition itself is neither true nor false, then it can be used as long as the actual situation is equivalent to one where both the presupposition and the sentence itself is true. This would explain the apparent embedded non-maximality that (37) permits.

In order for this to make sense, of course, homogeneity must not be a presupposition itself, and a presupposition failure's effect on assertability must be more direct and not mediated through the failure to be either true or false.

3.4 RELATIONS AND CUMULATIVE READINGS

As mentioned already in section 1.1.4, it is a common assumption that the relations denoted by lexical predicates in natural language are closed under pointwise mereological fusion. For convenience, the definition is repeated here.

DEFINITION 3.1. (Closure under Pointwise Fusion) For any relation R, its closure under pointwise fusion $^*R$ is the minimal relation such that for all a, b, c, d, if $R(a,b)$ and $R(c,d)$, then $^*R(a \oplus b, c \oplus d)$.[15]

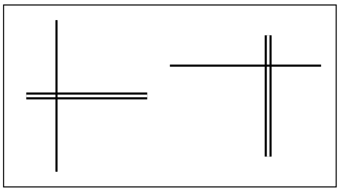
The reading that arises from this for a relational predication over two pluralities is sometimes called the *(weakly) cumulative reading*. For a sentence like (39), it is this: every boy must have kissed at least one girl, and every girl must have been kissed by at least one boy. Contrast this with the *double distributive reading*, which requires every boy to have kissed every girl.

(39) The boys kissed the girls.

The reason why pointwise closure under fusion is mildly suspect is that it seems to overgenerate weak readings. It predicts, for example, that (40) is true of the graphic in (41), because for every double line, there is a single line parallel to it and for every single line, there is a double line parallel to it.[16]

(40) The double lines are parallel to the single lines.

15 Note that this means that $R(a,b)$ entails $^*R(a,b)$, since this is just the special case with $a = c$ and $b = d$.
16 This kind of example is due to Schwarzschild 1996.

(41) 

Note that (42) *is* intuitively true of this display.

(42) The double lines are perpendicular to the single lines.

This kind of reading is generally regarded as not based on pointswise closure under fusion, but rather on a separate mechanism that uses some sort of contextual pairing (cf. Schwarzschild 1996). For a detailed discussion of such readings, and comparison of the two prominent approaches, see Winter 2000. The problem is, however, is not that this reading exists, but that the regular cumulative reading is completely unavailable.

This makes it worthwhile looking for an alternative to lexical closure under fusion, and I want to explore a view on which pointwise closure under fusion is not a feature of natural language relations at all. While I do not wish to argue definitively for the non-existence of the cumulative reading, the alternative strikes me as interesting to contemplate. With a proper theory of homogeneity, the doubly distributive reading is, of course, not simply false in situations where the cumulative reading is true, but undefined.[17] The idea is, then, that apparent cumulative readings, when they are available, are just the result of non-maximality. This, of course, requires showing that there is a plausible line of reasoning by which the doubly distributive reading could be weakened to something that looks like the cumulative reading.

There are, broadly speaking, three kinds of examples which the cumulative reading has been invoked to explain. The first is exemplified by (43) below.[18] Assume we are talking about the musicals *Oklahoma*, which was (one is told) jointly written by Rodgers and Hammerstein, and *On your toes*, which is the product of a collaboration between Rodgers and Hart. In this case, one is certainly inclined to judge the sentence as true.

(43) Rodgers, Hammerstein, and Hart wrote these musicals.

The scenario supplies (44a) and (44b) as primitive facts — this is simple a meaning of *write* that is collective with respect to the subject position. (44c) then follows by closure under pointwise fusion.

(44) a. $^{*}\mathit{wrote}(\mathit{Rodgers} \oplus \mathit{Hammerstein}, \mathit{Oklahoma})$
b. $^{*}\mathit{wrote}(\mathit{Rodgers} \oplus \mathit{Hart}, \mathit{On\text{-}your\text{-}toes})$
c. $^{*}\mathit{wrote}(\mathit{Rodgers} \oplus \mathit{Hammerstein} \oplus \mathit{Hart}, \mathit{Oklahoma} \oplus \mathit{On\text{-}your\text{-}toes})$

This kind of example seems plausibly explainable as the result of non-maximality weakening a stricter reading. Of course literal double distributivity does not

17 Cf. section 1.1.4.

18 This is an adapted version of an example passed down from Gillon (1987) through Champollion (2010).

apply here because *write* is (potentially) collective with respect to its subject. But it is easy to see what the relevant stronger reading is: both plays must have been jointly written by all three writers. If those are the truth conditions, however, the sentence is still not false in the present scenario.[19] Its acceptability, therefore, has to be due to non-maximality, and this would seem to be very plausible. In most contexts, what we care about is probably that all three writers were involved in the creation of the musicals, and that nobody else was. The precise configuration of the collaborations seems irrelevant for most purposes.

A serious obstacle is that this view cannot explain why *all* makes cumulative readings unavailable, at least for many speakers. (45) seems to only have a distributive reading which says that every student read (almost) all of the papers. It is not sufficient for every student to have read a paper and every paper to have been read.

(45) All the students have read these papers.

The approach explored here cannot explain this. *All* enforces maximality with respect to the subject position, so that the sentence is plainly false as soon as one student read no papers at all. However, when every student read at least some papers, which is the case in the cumulative scenario, the sentence is still true or undefined in just the same way as it would be without *all*. Thus an effect of *all* on the availability of cumulative readings does not follow.

The second kind of example that has been used to motivate closure under pointwise fusion is closely related the the above and involves indefinite plurals in object position. On the so-called *dependent plural* reading,[20] (46) doesn't require each of my friends to attend multiple schools, it only says that more than one schools is attended overall.

(46) My friends attend good schools.

The explanation is quite straightforward.[21] The sentence is assumed to have the simple logical form in (47), which just requires there to be some plurality of schools which stands in the relation ${}^{*}attend$ to the plurality of my friends.

(47) $(\exists x)(|x| > 1 \wedge {}^{*}\mathit{good\text{-}schools}(x) \wedge {}^{*}\mathit{attend}(\mathit{my\text{-}friends}, x))$

If cumulative readings do not exist, then this sentence is only true if there is a plurality of schools such that each of my friends attended each of them. An explanation in terms of non-maximality is not feasible here, as it is possible to explicitly add to (46) that they are not attending the same schools.

(48) My friends attend good schools, but not the same ones.

---

19 I am assuming here that conjunctions of proper names denote mereological sums, which participate normally in homogeneous predication. It is not clear what a theory on which conjunctions are not homogeneous would predict for these cases, or how, for that matter, closure under pointwise fusion would interact with it.

20 The term is due to de Mey (1981). For more discussion of this phenomenon, cf. chapter 5.

21 This is actually a slight, and at this point inconsequential, simplification of the proposals in the literature. See section 5.4 in chapter 5 for more details.

There is, of course, still the distributive reading of the sentence, which traditionally would be represented as in (49), where D is a distributivity operator.

(49) a. My friends DIST attend good schools.
b. $D(\lambda y.(\exists x)(|x| > 1 \wedge {}^{*}\textit{good-schools}(x) \wedge {}^{*}\textit{attend}(y, x)))(\textit{my-friends})$

In the logic of homogeneity from chapter 2, this reading is represented as in (50). D is the distributivity operator and π is the plural operator, which manipulates its argument so that it is undefined of atomic individuals, and $\mathcal{E}$ is an existential determiner and D the distributivity operator.

(50) $D(\lambda y.\mathcal{E}(\pi({}^{*}\textit{good-school}), \lambda x.{}^{*}\textit{attend}(y, x)))(\textit{my-friends})$

TRUE iff all of my friends attend more than one good school each.
FALSE iff none of my friends attends any good school.
UNDEFINED otherwise.

A situation where each of my friends attends exactly one good school, and more than one good school is attended overall, where the dependent plural reading of (46) is judged true, is therefore in the extension gap, and the judgment of truth could be due to non-maximality. But various other situations in which the dependent plural reading is not judged true are also in the extension gap, in particular, a situation where all of my friends attend the same school. A context that keeps these two apart would have to be such that it doesn't matter whether any of my friends attends more than one school, but it does matter that more than one school is attended overall. It is implausible that contexts would be like this so frequently as to conspire to bring out the dependent plural reading, such as it is, as something so recognisable that it could be mistaken for a separate reading.

Thus I conclude that if there is to be no closure under pointwise fusion of (lexical) predicates in natural language, then dependent plurals have to be explained in a completely different way, perhaps as more of a syntactic phenomenon.

The third, and, perhaps, most prominent kind of sentence that has been used to support the idea of closure under pointwise fusion is exemplified by (51a).[22] The sentence can be used in a situation where the total number of children born is seven; it does not necessarily require thirty-five births.[23] This is the only reading available once *between them* is added to the sentence.

(51) a. Five women gave birth to seven children.
b. Five women between them gave birth to seven children.

If *give birth to* is closed under pointwise fusion, then this can be straightforwardly explained by the logical form in (52), which just says that there is a plurality of five women and a plurality of seven children, and each of the women gave birth

22 This kind of example was first brought up by Scha 1981 and has since pervaded the literature on plural predication.

23 Of course, it does have such a distributive reading too, which may be used to level an accusation of reproductive excess against five women.

to at least one of the children, and each of the children was born by one of the women.

(52) $(\exists x)(\exists y)({}^{*}\mathit{women}(x) \wedge |x| = 5 \wedge {}^{*}\mathit{children}(y) \wedge |y| = 7 \wedge {}^{*}\mathit{give\text{-}birth\text{-}to}(x, y))$

On the view that only the doubly distributive reading exists, (52) would be nonsensical: it would require each woman to have given birth to each child, which is obviously impossible. This means that no explanation based on non-maximality is possible here, since it would require equivalence with an impossible situation.

However, there is some reason to doubt that closure under pointwise fusion is really the right explanation for this kind of sentence anyway (Benjamin Spector, p. c.). The cumulative reading requires that every part of the pluralities on both ends is somehow involved in the relation; in this particular case, for example, that each of the five women gave birth to a child. This predicts that (53) should be impossible, but it isn't.

(53) Twenty women gave birth to (only) seven children. What an infertile village!

It seems that such sentences only require that when aggregating over all women, the total number of children should be seven; but it does not preclude some women from contributing zero children. This is even weaker than the cumulative reading. It is not clear how exactly this meaning comes about, but the above discussion shows that sentences like (51a) are no conclusive refutation for the idea that cumulative readings don't actually exist and that relations in natural language are not lexically closed under pointwise fusion.

I cannot claim to have presented a complete picture of relational plural predication without pointwise closure under fusion, but I hope to have added an interesting perspective on the matter. Homogeneity and non-maximality may help to sustain such a view, but at this point, serious obstacles remain. I should note, though, that I have no particular stake in this: everything about the theory of homogeneity and non-maximality that I am defending is entirely compatible with the existence of lexical or even non-lexical closure under pointwise fusion.

3.5 EMBEDDED PLURALS AND LOCAL NON-MAXIMALITY

3.5.1 *The Scope of Quantifiers*

A plural in the scope of a quantifier gives rise to what looks like local non-maximality:

(54) a. Every student read the books.
 $\approx$ Every student did something that is, for current purposes, equivalent to reading all of the books.
 b. Exactly two students read the books.
 $\approx$ Two students did something that is, for current purposes, equivalent to reading all of the books, and all the other students did something that is equivalent to reading none of the books.

The theory I have presented predicts essentially this without recourse to any operation that applies locally in the scope of the quantifier. Recall from section 1.5 that a definite plural in the scope of a quantifier gives rise to an extension gap for the whole sentence. In the particular case of (54a), the truth and falsity conditions are as follows.

(55) Every student read the books.

TRUE iff every student read all the books.
FALSE iff at least one student read no books.
UNDEFINED never.

The sentence is therefore undefined in all the situations where it could intuitively be non-maximally true: when not every student read all the books, but every student read a number of books that is just as good as reading all of them. The appearance of local non-maximality comes about in the following way. (55) has the communicated meaning in (56), and it can, furthermore, only be used to address such issues where a single student reading no books at all would make a difference, that is to say, there can be no non-maximality with respect to the students. Consequently, any situation in which one student did something that is equivalent to reading no books at all is not a situation of which (56) is true.

(56) Something is the case that is, for current purposes, equivalent to all students reading all the books.

Analogous reasoning yields the desired results for all other quantifiers as well. It should be pointed out that it is perfectly possible for the exceptions to vary by students. If, for example, having read nine of ten books is just as good as having read all of them, then (54a) communicates just that all students read nine or more of the books; there is no requirement that all of them must have read the same nine books. This is a favourable prediction of the present theory that is not shared by other approaches to the phenomenon (cf. section 3.6).

3.5.2 *The Restrictor of Quantifiers*

As discussed in section 1.5.4, plural definites in a downward-entailing restrictor sometimes seem to have existential readings.

(57) Everybody who touched the statues was asked to leave.

In a situation where (57) would be uttered, it would seem that touching one statues is as bad as touching all of them. The seemingly existential reading of *touch the statues* could then be understood as extreme non-maximality. And indeed, effectively local non-maximality in the restrictor of quantifiers appears to be generally possible. A transitive sentence that is prone to being read non-maximally is (58). Sure it can be used to describe a situation where John likes most of his classmates, but is merely indifferent to some.

(58) John likes his classmates.

As expected of non-maximality, it is strange to mention an exception. If Mary is one of John's classmates, then (59a) has a contradictory flavour about it.

(59) a. #John likes his classmates, but he doesn't care for Mary.
b. John likes his classmates for the most part, but he doesn't care for Mary.

If this predicate is now embedded in the restrictor of a quantifier, then it apparently retains this non-maximal reading: (60) is naturally understood as saying that everybody who by and large likes his classmates is happy, not only that those people who like every single one of them are happy.

(60) Everybody who likes his classmates is happy.

If homogeneity projected from restrictors, we might have an extension gap to which the present theory of non-maximality could be applied and perhaps explain this. However, as discussed in section 1.5.4, there is good reason to believe that homogeneity does not project from restrictors, so that the literal meaning of (57) is as in (61).

(61) Everybody who touched the statues was asked to leave.

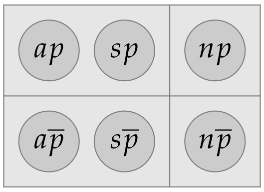
TRUE iff everybody who touched all the statues was asked to leave.
FALSE iff somebody who touched all the statues wasn't asked to leave.
UNDEFINED never.

If homogeneity doesn't project from restrictors, then (57) doesn't have an extension gap. Furthermore, non-maximality is normally a weakening of the literal meaning of a sentence, whereas what is found here is, viewed globally, a strengthening of (62a) to (62b).

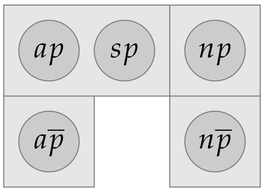
(62) a. Everybody who touched all the statues was asked to leave.
b. Everybody who touched any of the statues was asked to leave.

But perhaps a globalist explanation of the phenomenon can still be found. Assume that the current issue is such that touching any number of statues is equally bad. Thus, the cells of the current issue partition the common ground according to two questions: who touched any of the statues, and who was asked to leave? In order to make the argument, it is sufficient to look at just a single individual, resulting in a partition with four cells. It is also helpful to assume that there are only six words, distinguished by whether John touched all, some, or none of the statues, and whether he was punished by being sent out.[24] In the graphic below, worlds are designates by whether John touched all, some, or none of the statues (a, s, and n), and whether he was punished (p) or not ($\overline{p}$). The world, ap, for example, is the one in which he touched all the statues and was punished.

24 Of course it is absurd to quantify over a singleton domain with *everyone*, or to have a current issue that it so simplistic. I have merely isolated the features of the current issue that are relevant for the argument and ignore everything else.



The two worlds $a\overline{p}$ and $s\overline{p}$ are in the same cells of this issue, and John was punished in neither of them. This means that (57) violates the addressing constraint for this issue: the cell that contains $a\overline{p}$ and $s\overline{p}$ is compatible with both its negative and its positive extension. Faced with this situation, some implicature has to be drawn to fix the apparent violation of a pragmatic rule. If the partition is fixed — it is viewed as established that the number of statues touched doesn't matter — , then there is only one option: update the common ground with some proposition so that the issue, restricted to this new common ground, is such that it can be addressed with the sentence. This proposition has to be such that it either eliminates $s\overline{p}$ or $a\overline{p}$. If $a\overline{p}$ is eliminated, then only worlds where (57) is true are left, which means that the new assertion is suddenly uninformative. Thus, the only way is to accommodate a proposition that eliminates $s\overline{p}$: the proposition that if John touched only some of the statues, he was also punished. This leaves us with the updated issue in (63), which can legitimately be addressed with (57).



If there are more individuals than just one to be taken into account, the picture becomes less easily visualised, but the proposition that needs to be accommodated is (63).

(63) If anyone touched only some of the statues, they were punished.

One could thus attempt to explain the existential reading of the plural in (57) as due to an implicature triggered by the addressing constraint; essentially a kind of relevance implicature.

However, this view is faced with several challenges. The first is that universal quantifiers in natural language tend to entail that their restrictor is non-empty, and indeed (57) conveys that someone did touch at least some of the statues. According to the semantics assumed, however, it should entail that someone touched *all* of the statues.

A second challenge is posed by the fact that (64) has the same truth and falsity conditions as (57), but does not allow an existential reading for *all the statues*.

(64) Everybody who touched all the statues was aked to leave.

Furthermore, there is a problem with quantifiers that are non-monotonic with respect to their restrictor restrictor. It is predicted that (65a) always entails (65c) (since that has the same literal truth conditions) and cannot communicate just (65b).

(65) a. Most people who touched the statues were asked to leave.
b. Most people who touched any of the statues were asked to leave.
c. Most people who touched all of the statues were asked to leave.

It is not clear to me what the empirical situation actually is, but it seems quite possible that (65c) is not actually an entailment of (65a) when touching the statues is understood non-maximally. (65a) would then plausibly be judged true if most people who touched any of the statues were asked to leave, but it just so happened that those few who touched every single one of them were among the minority who weren't asked to leave.

The deathblow for any attempted globalist explanation of non-maximality in restrictors comes from definite descriptions. In principle, the globalist story that applies to *every* can also be used to explain why (66a) seems to entail both (66a) and (66b).

(66) a. The students who touched the statues were asked to leave.
b. The students who touched any of the statues were asked to leave.
c. The students who touched all of the statues were asked to leave.

However, now there is the further question of what the *reference* of the definite description in (66a) is. According to the above theory, the reference has to be the sum of all students who touched *all* of the statues. Unless non-maximality is actually computed locally, with the result fed to the definite article as an argument, this is the only possibility. It would seem that this is wrong. For one thing, (66a) doesn't presuppose that any student touched all of the statues, or even that all statues were touched.[25] Thus is must be concluded that non-maximality is truly local: the meaning of (66a) is actually (67). No globalist pragmatic theory can explain this.

(67) The students who did something that is, for current purposes, equivalent to touching all of the statues were asked to leave.

The idea that implicatures can be computed locally, however, is by no means a new one. It has famously been argued for scalar implicatures. The standard version of the grammaticalist view of scalar implicatures is that there is an operator that computes them, and that this operator can be inserted locally (Chierchia et al. 2012) It is quite easy to define an operator that would locally compute the quality implicatures that lead to non-maximal readings, and unlike the exhaustivity operator for scalar implicatures, that operator is even wholly compositional. The basic version would take an issue — which I here conceive of as an equivalence relation between worlds — and a proposition as its arguments and return a proposition.

(68) $[\![\textsc{non-max}]\!] = \lambda I.\lambda p.\lambda w.\exists w' : p(w') \wedge I(w, w')$

25 The latter would follow on a cumulative reading of the predication in the relative clause. On such a reading, the definite would refer to the maximal plurality of students that is such that each of them touched one of the statues and each statue was touched by one of them.

It is obvious how to define a polymorphic version that maps predicates to predicates, so that it can be applied in the restrictor of a quantifier.

3.6 COMPARISONS

3.6.1 *Brisson 1998*

Brisson (1998, 2003) presents an account of the possibility of non-maximal readings based on Schwarzschild's (1996) theory of covers. Greatly simplified, we may illustrate the theory thus: when a predicate is combined with a plurality, a contextually supplied function C intervenes which maps that plurality to a subplurality of itself. Thus, the meaning of (69a) is (69b), for some contextually supplied C.

(69) a. The professors smiled.
b. $[\![\text{smiled}]\!](C([\![\text{the professors}]\!]))$

When C is the identity, then we obtain a maximal reading. If the context supplies a C that maps the professors to a proper subplurality of them, then we get a non-maximal reading. The way that *all* now removes the possibility for non-maximal readings is by forcing C to be the identity function regardless of the context, which it accomplishes through some unspecified action-at-a-distance mechanism. No provision is made for the, as it were, super-maximal readings with collective predicates that were discussed in subsubsection 3.6.3.2, but it seems possible in principle to allow for them by changing what kinds of functions C be (i. e. it could map a plurality not only to a subplurality, but to any overlapping plurality).

Brisson does not explore the rules that govern the choice of C for a given context, nor does her account incorporate homogeneity. It is difficult to see whether the theory could be developed into something of similar or wider coverage than what we have presented, and we cannot explore this question here, so a meaningful comparison is hardly possible.

3.6.2 *Burnett 2013*

Burnett (2011b) defines a semantics for predicate logic with plurals — though only for unary distributive predicates — in which a sentence is assigned three kinds of truth-conditions, in an adaptation of a system originally developed under the name of *Tolerant, Classical, Strict* by Cobreros et al. (2012) for vague predication. Classical truth is two-valued and basically gives plural predication universal truth-conditions and complementary (existential) falsity conditions. Classical truth, however, is only used to derive the two other notions: tolerant truth and strict truth. To do this, a relation $\sim$, parameterised by predicates, is added to the model. For any predicate P, the binary relation $\sim_P$ between individuals is reflexive and symmetric, but not necessarily transitive. Assume that for any individual a, $\underline{a}$ is a name for that individual in the object language. Then the truth definitions are as follows:

(70) **Tolerant and Strict Truth**
$M \vDash^t P(\underline{i_1})$ iff $\exists i_2 \sim_P i_1 : M \vDash^c P(\underline{i_2})$
$M \vDash^s P(\underline{i_1})$ iff $\forall i_2 \sim_P i_1 : M \vDash^c P(\underline{i_2})$

While classical falsity is simply the negation of classical truth, the tolerant and strict logic have their falsity conditions defined in terms of each other:

(71) **Tolerant and Strict Falsity**
$M \vDash^t \neg\phi$ iff $M \nvDash^s \phi$
$M \vDash^s \neg\phi$ iff $M \nvDash^t \phi$

The relation $\sim$ is not analysed any further, but given primitively in the model. However, it is clear what notion it is meant to reflect: if $i_1 \sim_P i_2$, then $P(\underline{i_1})$ and $P(\underline{i_2})$ are, for present purposes, equivalent. This will be the case if i_1 is a plural individual that contains some irrelevant exceptions that do not fulfill P, while i_2 is i_1 minus those exceptional individuals. That $\sim_P$ is assumed to be symmetric has a problematic implication. Assume that i_1 contains some exceptions to P, while i_2 is the maximal subgroup of i_1 that contains no exceptions; and furthermore, that $i_2 \sim_P i_1$. Then $P(\underline{i_1})$ is tolerantly, but not strictly true, but $P(\underline{i_2})$, despite the fact that i_2 contains no exceptions, is also not strictly true, because there is i_1, with which it stands in the relation $\sim_P$ and of which P is not classically true. However, this is easily remedied by making $\sim_P$ antisymmetric, which I see no reason not to do.[26]

Assume that all the details are worked out and things are as they should be: a sentence $P(\underline{a})$ is strictly true if all members of a are P, and tolerantly true if a contains only irrelevant exceptions to P. There are then some puzzling consequences, which may be adequate for vagueness, but seem odd for plurals. If tolerant truth is what enables non-maximal readings, then it is predicted that a sentence and its negation are always equally appropriate on a non-maximal reading, which is clearly absurd. See also section 1.7.2 for why such, while perhaps appropriate for vagueness, is problematic for plurals.

Furthermore, polarity is only predicted for the strict interpretation, and even then, it is a weird kind of polarity: a predicate would be strictly false on individuals that it is not tolerantly true of. That is, as soon as we find a point where we are definitely not inclined to call the sentence true, we should immediately call it false. Again, this doesn't seem to be appropriate for plurals. If exactly half the boys went swimming, (72) is definitely not true; but it's definitely not false, either.

(72) The boys went swimming.

Burnett's (2011a) theory is the only one that we are aware of (besides our own) that links the effects that *all* has on homogeneity and non-maximality. In this, it relies on the peculiar features of the TCS system. Burnett proposes that the truth-conditions for predication with *all* are very simple:

26 In fact, Burnett (2014) argues on psychological grounds that a symmetric $\sim$ is probably not desirable in the analysis of vague adjectives, either.

(73) $M \vDash^t P(all\ \underline{a})$ iff $M \vDash^s P(\underline{a})$
$M \vDash^s P(all\ \underline{a})$ iff $M \vDash^s P(\underline{a})$

That is, *all* simply sets the tolerant truth conditions to be identical to the strict ones. This makes *all* remove both polarity and non-maximality in one fell swoop, since by (71), $P(all\ \underline{a})$ is strictly false iff it is not tolerantly true, and by (73) it is tolerantly true iff it is strictly true, so it is strictly false iff it is not strictly true. This elegantly connects the polarity-removing and precisifying effects of *all*, and it also sits well with the fact that *all* is not an alternative to the definite article, but is added, as it were, on top of a definite DP. However, as we saw above, the polarity in the system is of an odd sort that doesn't appear to be quite what we see with plurals, and if it were to be replaced by something more appropriate, the connection that Burnett's system establishes between polarity and non-maximality would be broken.

3.6.3 *Malamud 2012*

Malamud (2006, 2012), too, starts from the idea that plural sentences are interpreted with reference to a partition on the set of possible worlds that formalises an intuitive notion of the current purposes of the participants of the conversation, in doing so, presents the first theory to say something substantial about the way in which non-maximal readings depend on context. However, apart from this commonality, Malamud's theory has a very different architecture from the one I have presented. In the following, I will put forward a critique of it substantiating the following four points.

1. Malamud has to assume a non-compositional interpretation procedure for definite plurals.
2. The theory makes the same predictions as ours for distributive predicates, but overpredicts not only non-maximal, but existential readings with collective predicates.
3. The theory doesn't deal with embedded plural definites very well.
4. The theory does nothing to link homogeneity and non-maximality.
5. The theory has nothing to say about how the addition of *all* prevents non-maximal readings.

3.6.3.1 *The Interpretation Procedure*

To first see intuitively how Malamud's interpretation procedure[27] procedure works, consider, as the simplest case, a sentence ϕ containing one definite plural DP α being used to address a question Q. The intuitive idea is that you first form various alternatives of ϕ which are obtained by replacing α by a different definite

27 We should note that, in an attempt to improve transparency and legibility, we have chosen to present Malamud's theory in a manner quite different from what is found in the original paper. We would like to point out that the final definition of Malamud's operator in (58) on p. 38 takes expressions, and not denotations, as its arguments and cannot be reduced to a compositional operator, even though its predecessor in (51) is compositional.

description whose denotation is a mereological part of α's denotation, so if the sentence is (74a), then (74b) and (74c) are, informally, some of these alternatives.

(74) a. The professors (= Jones, Brown, and Smith) smiled.
b. Jones and Brown smiled.
c. Smith smiled.

Then you take, from this set of alternatives, those which are most relevant to the question Q, and form the disjunction of them. This disjunction is the final meaning of the sentence.

In defining the procedure formally, we shall write $[\![\cdot]\!]$ for the usual denotation function, and $[\![\cdot]\!]^*$ for the final meaning of the sentence after Malamud's procedure has been applied. The latter has the following form:

(75) $[\![\phi]\!]^* := \lambda w.\exists p : p \in \max_Q(\mathrm{Alt}_w(\phi)) \wedge p(w)$

Formally, the set of propositions $\mathrm{Alt}_w(\phi)$ can be defined as follows:[28]

(76) $\mathrm{Alt}_w(\phi) := \{[\![\phi[\xi/\alpha]]\!] | [\![\xi]\!](w) \sqsubseteq [\![\alpha]\!](w) \text{ and } [\![\xi]\!] \text{ is constant}\}$

To obtain this set, we have to perform several steps.

1. Take the extension of the plural DP α in the world w. (α itself denotes an individual concept, i. e. a function from worlds to individuals.)
2. Collect all the individuals x_i that are mereological (not necessarily atomic!) parts of $[\![\alpha]\!](w)$.
3. For every such individual x_i, take an expression ξ_i which denotes a constant individual concept that in all words has x as its extension.[29]
4. Then we take the sentence ϕ and replace α with the various ξ_i, yielding a collection of sentences.
5. Take the denotations of these sentences and collect them.

The set of propositions that this procedure yields is $\mathrm{Alt}_w(\phi)$. If the professors in w are Smith, Brown, and Jones, for example, and the sentence in question is *The professors smiled*, then $\mathrm{Alt}_w(\phi)$ is the following set of propositions:

$$\begin{aligned}&\{[\![\text{Smith smiled}]\!], [\![\text{Brown smiled}]\!], [\![\text{Jones smiled}]\!],\\ &[\![\text{Smith and Brown smiled}]\!], [\![\text{Brown and Jones smiled}]\!],\\ &\qquad\qquad\qquad [\![\text{Smith, Brown and Jones smiled}]\!]\}\end{aligned}$$

28 Assuming λ-abstraction in the object language, there is an alternative formulation that does not presuppose that we have names for all the individuals involved:

(i) $\mathrm{Alt}_w(\phi) := \{[\![\lambda\xi.\phi[\xi/\alpha]]\!](x) | x(w) \sqsubseteq [\![\alpha]\!](w) \text{ and } x \text{ is constant individual concept}\}$

29 For the reader who wishes to compare our reconstruction with Malamud's presentation, we note that the definitions of *REL* in (51) and (58) clearly select individuals, not individual concepts. Thus, the writing $g(w)$ in (55b) and (56b) must be assumed to be an oversight. The same holds for p instead of $p(w)$. Thus, (55b) should read, with corrections in boldface:

$$\lambda w.\exists p[p \in \{\mathbf{g}\ \textit{is open} | \mathbf{g} \in REL(DeP)(win)(open)(w)\} \wedge \mathbf{p}(\mathbf{w})]$$

Analogous corrections apply to (56b).

Going back to (75), we also need to know what $\max_Q$ does. $\max_Q(\mathrm{Alt}_w)$ designates the subset of $\mathrm{Alt}_w(\phi)$ that consists of those propositions that are maximally relevant to the question Q, where relevance is assessed by counting the number of cells in Q that a proposition eliminates. $[\![\phi]\!]^*$ then contains the world w if the disjunction of all these maximally relevant propositions is true in w, and analogously for a different word u if it makes one of the most relevant propositions based on the set of professors in u true, etc.

This definition has disastrous consequences in the general case. For the constitution of the set of professors may vary from world to world; Smith may be a professor in some worlds, but not in others, and the hearer of an utterance of *The professors smiled* may have no idea which particular individuals the professors were. Thus, there will be worlds where Smith is a professor in all cells of the current issue, and worlds where he isn't. The proposition that Smith smiled, therefore, is simply irrelevant: it doesn't eliminate a single cell; and similarly for all other propositions that are about particular individuals. Whenever such a situation arises, all propositions are equally relevant. This means that $[\![\phi]\!]^*$ contains w if just one of these professors (the professors in w) smiled, and contains u if one of the professors in u smiled, etc. Thus, in general, when the extension of the plural DP is not known to be the same particular group in all worlds, Malamud's procedure yields an existential reading for plural definites. In order to get an even close to maximal reading, the hearer has to know which particular individuals most of the professors were (and furthermore we have to assume that the question Q partitions not all possible worlds, but is restricted to, say, the common ground). This obviously cannot be right, but as will shortly become apparent, it is not an unsolvable problem.

For the moment, let me set it aside by assuming that *the professor* has the same extension in all worlds, i. e. that the professors have been established to be a particular group of individuals. It then turns out that the theory cannot distinguish between relevant and irrelevant exceptions. In section 3.3.2, I argued that it is important what the professors who don't smile do instead. In Malamud's system, such situations lead to overly strong readings. Assume that the current issue is $Q = \{i_1, i_2, i_3\}$. i_1, corresponding to a favorable reception of Sue's talk, contains worlds where all professors smile, and also those worlds where all professors except Smith smile and Smith doesn't anything to overtly indicate displeasure. i_2 contains, among others, worlds where all the professors except Smith smile and he displays anger. i_3 contains only worlds where less than two professors smile. Then the proposition that Jones and Brown smiled eliminates only i_3, whereas the proposition that all three professors smiled eliminates i_3 and i_2, consequently being more relevant. What results is a maximal reading: all three professors smiled. In general, as soon as there is at least one way in which an individual could be a relevant exception, it is precluded from being an exception in any way at all.

Both of these problems can be fixed by restating the theory in terms of individual concepts instead of individuals, which furthermore simplifies its presentation. Take again the sentence ϕ with the definite description α. We then compute, world-independently, a set of propositions, let's call it $\mathrm{Alt}(\phi)$.

(77) $\text{Alt}(\phi) := \{[\![\phi[\xi/\alpha]]\!] | \forall w : [\![\xi]\!](w) \sqsubseteq [\![\alpha]\!](w)\}$

These propositions are again the meanings of variants of ϕ where α is replaced by another expression that denotes an individual concept. But this time, ξ need not be constant — we merely require that its extension in any world be a part of the extension of α in that world.[30] Setting aside matters of definedness, ξ could be something like *the female professors*.

The effective meaning of ϕ is then the disjunction of the maximally relevant propositions in that set:

(78) $[\![\phi]\!]^* := \bigvee \max_Q(\text{Alt}(\phi))$

Now it doesn't matter if the set of professors varies from world to world — we can have the individual concepts in the alternatives vary with it. To see how this takes care of the problem concerning the identity and activities of exceptions, take the individual concept a such that $a(w)$ denotes $[\![$the professors except Smith$]\!](w)$ if Smith deviates from smiling in an unimportant manner in w, and otherwise denotes $[\![$the professors$]\!](w)$. This individual concept is obviously a part of $[\![$the professors$]\!]$ by the above definition. Then $\lambda w.[\![\text{smiled}]\!](w)(a(w))$ is in the set of candidate propositions, and, by construction, it is one of the maximally relevant propositions; so Smith ends up being permitted as an exception depending on what he does instead of smiling, replicating the results of our theory.

3.6.3.2 *Non-Maximality with Collective Predicates*

On the empirical side, there is a dubious prediction of Malamud's account that arises specifically with collective predicates. Assume a scenario in which all the children together, the boys and the girls, performed *Hamlet*. (79) is not be true in that case, because the predicate *performed Hamlet* doesn't hold of a subplurality of the children; it does not, after all, mean *participate in a performance of Hamlet*.

(79) The boys performed *Hamlet*.

Conversely, (80) is not true if *only* the boys formed a circle, whereas the girls didn't participate and did something else instead.

(80) The children performed *Hamlet*.

If this is so, then Malamud's machinery overpredicts existential readings for collective predicates. Assume that the current issue contains three cells: i_1, where all the children together performed the play, i_2, where some proper subgroup of the children performed the play, and i_3, where there was no performance at all, and that further the denotation of *the children* is the same group c_1 in all worlds. Let c_2 be any proper subgroup of the children. The alternative propositions, from which the most relevant ones are to be chosen, will contain, among others,

30 To be completely precise, we have to take into account the fact that definite descriptions are not defined for all worlds; thus, the above definition should be understood as involving quantification only over those worlds in which both concepts are defined, and we may further want to require that $[\![\xi]\!]$ should have the same domain as $[\![\alpha]\!]$.

the proposition p_1 that c_1 performed *Hamlet*, and the proposition p_2 that c_2 performed *Hamlet*. p_1 is true only in i_1, and thus eliminates two cells — i_2 and i_3. p_2 is true in some worlds in i_2, but not in any worlds in i_1 or i_3, and hence eliminates two cells as well. Thus, p_1 and p_2 are equally relevant, and the eventual meaning of (80) will contain them as disjuncts. Since c_2 is any subgroup of the children, what results is simply an existential statement: that some (proper or improper) subgroup of the children performed the play — in spite of the fact that by stipulation, it matters for current purposes whether it was all or only some of the children that staged the performance. This is clearly inappropriate.

Worse yet, one can even construct a scenario in which $[\![(80)]\!]^*$ eliminates no cell from the current issue at all. Assume that there are some worlds in i_3 where a subgroup c_3 of the children performed the play, and that c_3 did not perform the play in any world in i_2 or i_1. Then the disjunction that is $[\![(80)]\!]^*$ contains as disjunct, among other things, the propositions that c_1 performed *Hamlet*, that c_2 performed *Hamlet* (which is only true in worlds in i_2), and that c_3 performed *Hamlet*, and is thus compatible with all three cells of the current issue.

This problem arises with collective predicates due to the fact that they are not upward-monotonic with respect to the algebra that is the domain of individuals. For distributive predicates, the proposition p_2 would always be true throughout i_1, and so eliminate fewer cells than p_1 (thus being less relevant and not occurring in the disjunction that is the final interpretation of the sentence in question) unless it is indeed, for current purposes, equivalent to p_1, in which case it would eliminate equally many cells.

The theory can be repaired in a way that makes it, as far as the predicted non-maximal readings for plurals are concerned, equivalent to the one presented in this paper. The reason for the above prediction is that we chose the most relevant alternative propositions, not the ones that identify the same cell as the maximal interpretation. If the latter rule is used instead, the theory makes equivalent predictions to ours about the conditions under which non-maximality is possible.

3.6.3.3 *Embedded Plurals*

Malamud's theory makes predictions for embedded definite plurals. Assume that there are three books a, b, and c. Then the candidates generated for (81a) are as in (81b).

(81) a. Every student read the books.
b. $[\![$Every student read $a]\!]$, $[\![$Every student read $b]\!]$, $[\![$Every student read $c]\!]$, $[\![$Every student read a and $b]\!]$, . . .

The final meaning of (81a) is going to be the disjunction of some of the propositions in (81b), which allows for some measure of non-maximality. If, for example, reading two books is as good as reading three for current purposes, then the result is (82).

(82) $[\![$Every student read a and $b]\!] \vee [\![$Every student read b and $c]\!]$
$\vee [\![$Every student read a and $c]\!] \vee [\![$Every student read a, b, and $c]\!]$

Importantly, however, (82) still entails that every student read the same books. It cannot be the case that one student read *a* and *b*, but not *c*, while another read *b* and *c*, but not *a*, despite the fact that, by stipulation, reading two books was as good as reading three. In general, Malamud's theory predicts that when a plural embedded under a quantifier is read non-maximally, the exceptions are the same for all witnesses of the quantifier. This strikes me as undesirable.[31]

3.6.3.4 *Homogeneity*

An approach like Malamud's, due to its fundamental structure, furthermore does nothing to link non-maximality and homogeneity. The theory works for negated sentences only if homogeneity is implemented in the underlying logic.[32] Assume that the professors are Brown, Jones, and Smith in all worlds, and we want to interpret (83).

(83) The professors didn't smile.

Then the set of propositions from which we need to pick the most relevant ones contains, among others, these:

(84) ⟦Jones didn't smile⟧, ⟦Jones and Smith didn't smile⟧, ⟦Smith didn't smile⟧, ⟦Brown didn't smile⟧, ⟦Jones, Brown and Smith didn't smile⟧, . . .

If we assume no homogeneity for these sentences, so that they have their classical truth conditions — so that *Jones and Smith didn't smile* is true as soon as either of them failed to smile — , then we have a problem: the final interpretation of (83) is to be a disjunction of such propositions, but no such disjunction will ever amount to the proposition that none of the professors smiled. Indeed, it would be impossible for a negated plural sentence to entail that more than one individual in the plurality failed to satisfy the predicate! Only if we assume that the propositions in the set of alternatives say of various subpluralities of the professors that *all* members of those pluralities failed to smile — i. e. if the underlying logic has homogeneity effects — do we get the desired reading, since then we pick the most relevant ones from among propositions like the following:

(85) ⟦Jones didn't smile⟧, ⟦Neither Jones nor Smith smiled⟧, ⟦Smith didn't smile⟧, ⟦Brown didn't smile⟧, ⟦Neither Jones nor Brown nor Smith smiled⟧, . . .

3.6.3.5 *Conclusion*

Malamud 2012 is an important improvement over previous discussions of non-maximality in that it spells out explicitly how a notion of current purposes of the conversation influences the availability of non-maximal readings. However, the foregoing discussion has identified a number of shortcomings in the theory, on which grounds I content that the approach presented in this chapter is more promising.

31 I thank Benjamin Spector (p. c.) for discussion on this point.

32 The discussion of example (64) on p. 34 in the paper is liable to be read as suggestive of the contrary, as the assumption of homogeneity in the underlying logic is not made explicit.

3.7 CONCLUSION

The theory I have proposed in this chapter conceives of non-maximal readings of sentences with definite plurals as the result of the interplay between the trivalent semantics of sentences with a homogeneity-induced extension gap and certain pragmatic principles. The particular proposal assumes a weakened maxim of quality, which says that a sentence may be used to describe a world in which it is not literally true as long as that world is, for current purposes, equivalent to a world where the sentence *is* literally true. An additional constraint allows a sentence to be used only if it matters for current purposes whether it is true or false, which has the effect of precluding sentences from being used in situations where they are literally false. These two principles were formalised and the intuitive notion of *current purposes* was operationalised as a partition of the set of possible worlds.

A consequence of this is that only sentences with an extension gap can be used non-maximally, which explains the correlation between homogeneity and exception tolerance (on which see also chapter 7), and which, in particular, explains the maximising/"slack regulating" function of *all* as a consequence of its semantics alone: as discussed in chapter 1, *all* removes (most of) the extension gap that is due to homogeneity, and thereby one of the ingredients necessary for non-maximal readings.

Based on the picture of homogeneity presented in chapter 1, I have identified numerous welcome predictions of this approach, but also noted some remaining puzzles (sections 3.3 through 3.5). Finally, the theory was compared and argued to be superior to previous approaches to the phenomenon (section 3.6).

An open issue that has to be acknowledged is the fact that while *all* clearly reduces the tolerance for exceptions, and often eliminates it completely, it is not always entirely impossible to use *all* despite the presence of some exceptions. (86) probably does not really require every single person at the party to be very happy.

(86) All the people at the party were very happy.

This is not, to my knowledge, a phenomenon that any existing theory can explain, and it remains to be seen whether the suggested approach can be improved in a way that allows for this possibility. It is, of course, also quite possible that such uses of *all* are just a form of hyperbole that is a completely different phenomenon from non-maximality. At any rate, this is a question for further research.

3.A APPENDIX: EXTENSION TO NUMERALS

Numerals are one of the most well-known kinds of expressions that are used imprecisely (Krifka 2002, 2007), that is, they are used to describe situations of which they are not, in some sense, strictly true.[33] It is attractive to trace such

33 I consider proportional quantifiers like *half (of)* and *three quarters of* to be numeral-containing expressions for these purposes.

imprecise uses to a common source, but there seems to be a very serious obstacle to generalising our proposal to numerals: in our derivation of non-maximal readings, we relied essentially on the polarity property of plurals, which numerals *prima facie* don't have.

Let us first look at Krifka's influential theory of how imprecision arises with number words. It is an established fact that some number words are vastly more frequent than others, and it has been surmised that the corresponding concepts are, in some sense, cognitively more prominent; for example, *a hundred* is much more than *ninety-seven* (Dehaene & Mehler 1992). From this, we can derive different number scales, which include numbers up to a certain level of fine-grainedness. Which numbers are prominent depends on the purpose at hand; if we want to give the time in minutes, 0, 30, 60, and 90 are most prominent, whereas in the general domain of counting, 50 is usually considered more prominent than 60, and 100 is certainly more prominent than 90.

For time measurement in minutes, Krifka assumes a collection of scales like the following.

(87) $s_a : 0 - 60 - 120 - \ldots$
$s_b : 0 - 30 - 60 - 90 - 120 - \ldots$
$s_c : 0 - 15 - 30 - 45 - 60 - 75 - 90 - \ldots$
$s_d : 0 - 10 - 20 - 30 - 40 - 50 - 60 - \ldots$
$s_e : 0 - 5 - 10 - 15 - 20 - 25 - 30 - 35 - \ldots$
$s_f : 0 - 1 - 2 - 3 - 4 - 5 - \ldots$

These scales can be ordered in an obvious manner: a scale s_α is a coarsening of a scale s_β (s_β a precisification of s_α) iff s_β contains all the values on s_α, and some more. Each of these scales has an associated measure function (μ_a, μ_b, etc.) that measures, say, the duration of an event by mapping that event to a point on the scale. A natural language sentence involving a numeral is understood as a statement about the value of a measure function for some object; but it is ambiguous, or underspecified, with respect to which measure function is being used — at which level of precision the discourse is being held — and hearers have to reason about this issue.

It is obvious that there must be some relation between the measure functions associated with scales. For example, it is unthinkable that μ_b could map an event to 60, while the coarser measure function μ_b maps the very same event to 120. Krifka proposes the following rule:

(88) For any individual/event x, if s_α is a coarsening of s_β, then $\mu_\alpha(x)$ is that value on s_α which is closest to $\mu_\beta(x)$.

For example, if μ_f maps the event e to 53, then μ_d has to map it to 50, because that is the value on s_d which is closest to 53. Analogously, μ_c maps it to 60. Note that μ_c is not a coarsening of μ_c (nor the other way around), so that μ_c is allowed to map e to 60 even though 45 would be closer to its μ_d-value 50. One may therefore think of the points on scales as if they were particular intervals of points on more precise scales. For instance, the point 30 on s_c corresponds to the interval $[23, 37]$ on s_f.

Note that although Krifka does not discuss this, there is no presupposition here that infinitely precise measurement is possible. Rather, we merely need to assume that there is one maximally precise scale on which an individual or event can be measured, and that automatically gives us its values on all coarser scales. This is welcome, as it seems awkward to have to assume that every event has an absolutely precise duration or happens at an absolutely precise point in time.[34] What *is* the precise moment at which John's running ended? It seems that this is only specifiable up to a certain level of precision and then becomes unclear due to vagueness. A theory that assumes that arbitrarily precise measurement for everything is possible is thus very much akin to the ill-liked epistemic theory of vagueness,[35] which posits that there is always a fact of the matter as to whether an individual falls under a concept or not, but we just don't always know where the cut-off point is.

Krifka now derives the preference for interpreting round numerals imprecisely by assuming that hearers do probabilistic reasoning about which scale the speaker used to derive at the reported value. For this purpose, one needs to start with a prior probability distribution that gives, for all numbers, the prior probability of that number being the correct value on the most precise scale, and the prior probability of a given scale being used. For illustration, let us assume that the most precise scale on which the event of John's running can be measured is s_f, and that the distribution is uniform over the values between 1 and 60 on it. Thus, for any $n \in [1, 60]$, the probability $P(n|s_f)$ is $\frac{1}{60}$. Furthermore, the probability distribution over the scales in (88) is also uniform, so that $P(s_a) = P(s_b) = \cdots = \frac{1}{6}$. From this, we can derive the conditional probabilities of all the values given all the other scales. For example, since 45 on s_e corresponds to the interval $[43, 47]$ on s_f, the probability $P(45|s_e)$ is equal to $P([43, 47]|s_f)$, which is $\frac{5}{60}$.

Upon hearing a sentence like (89), the hearer can now applying Bayesian reasoning to deduce which scale is probably being used simply by updating with the fact that *fourty-five* was uttered (and that the speaker is not misrepresenting reality).

(89) John ran for forty-five minutes.

Fourty-five occurs only on s_c, s_e, and s_f, so we can immediately discard the other three scales as contenders. What we need to look at is the prior probabilities of *fourty-five*, given our three remaining candidates s_c, s_e and s_f.

$$P(\textit{fourty-five}|s_f) = \frac{1}{60}$$
$$P(\textit{fourty-five}|s_e) = P([43, 47]|s_f) = \frac{5}{60}$$
$$P(\textit{fourty-five}|s_c) = P([38, 52]|s_f) = \frac{15}{60}$$

34 Presumably, an absolutely precise value would be a point in the continuum, i. e. in $\mathbb{R}$.
35 Cf. Williamson 1994 for the original proposal, and everybody else's work for disapproval.

By Bayes's rule, it now follows that the posterior odds for $s_c : s_e : s_f$ are $15 : 5 : 1$. It is therefore very unlikely that s_f was being used, and there is a chance of $3 : 1$ that s_c, rather than s_e, is the scale that the speaker based her assertion on. Thus, a hearer will, when faced with a round number, assign a high probability to the hypothesis that a relatively coarse-grained scale was is employed. If instead the numeral she heard was *five*, there is a much lower upper bound on coarse-grainedness: the least detailed scale on which *five* even occurs is s_e.

Krifka does not consider this issue, but one would presume that the semantic effect of *exactly* is to somehow disambiguate the sentence by indicating that the scale being used is the most precise one that is applicable.

The picture presented above can be reconstructed to a close approximation in a manner that unifies imprecision with number words and non-maximality with plurals (as well as the precisifying effects of both *all* and *exactly*). In order to achieve this, we need slightly different correspondence rules that connect the measure functions associated with different scales. The first one is already implicit in Krifka's approach, too:

(90) PRESERVATION OF MEASUREMENT
For any individual/event x, if s_α is a coarsening of s_β, and $\mu_\beta(x)$ is on s_α, then $\mu_\alpha(x) = \mu_\beta(x)$.

More informally, if an individual is assigned a value n on a scale, then it is assigned the same value n on all coarser scales that contain n. For example, if the event is assigned a duration of 60 (minutes) by μ_b, then it must be assigned the same value by μ_a.

Now assume that μ_c assigns the duration of 45 to the event of John's running. What should the value of μ_b for that event be? If we followed Krifka and chose the closest value that is on s_b, then we would never obtain a significant extension gap that could serve as the basis for an imprecise predication. Therefore, we assume a different principle: if the value of the more precise measure function is not available on the coarser scale, then the coarser measure function has to remain undefined.

(91) ANTI-ALTERATION OF MEASUREMENT
For any individual/event x, if s_α is a coarsening of s_β, and $\mu_\beta(x)$ is not on s_α, then μ_α is not defined for x.

We take it that any statement about the value of a measure function for an argument is neither true nor false if the measure function is not defined for that argument. Thus, given the above scenario, (92) is true if interpreted with respect to s_c, and neither true nor false if interpreted with s_b (or s_a).

(92) John ran for forty-five minutes.

There are also pairs of scales in which neither is a coarsening of the other; s_c and s_d, in our case. What Krifka seems to assume implicitly, we state as a principle here:

(93) NON-COMPETITION OF MEASUREMENT
For any individual/event x, if s_α is not a coarsening of s_β, nor *vice versa*, then either $\mu_\alpha(x) = \mu_\beta(x)$, or at least one of these two measure functions is not defined for x.

In general, this amounts to uniqueness of the most precise measure function that is defined for the individual/event.

So now we have extension gaps that we can use to derive imprecision. Take, again, (92). If it is interpreted with s_c, then it is true if the most fine-grained measure-function that is applicable to the event (whichever that may be) maps it to 45, and false if that measure function maps it to any other multiple of 15. In the case of s_e and s_f, the truth-conditions stay the same, but the falsity conditions include other multiples of 5 and 1, respectively, thus the extension gap becomes narrower. s_a, s_b and s_d don't come into play because the sentence can never be true with respect to them anyway.

This means that, no matter which scale is used, the sentence can always be said of a situation that is, for current purposes, equivalent to one in which the most precise measure function yields 45. Thus, in order to figure out its communicated content, one need not even concern oneself with the question of which scale the speaker has in mind.[36]

However, the choice of scale influences what issue can be addressed. s_c requires that there is no cell which contains worlds where the most precise measure function yields 45 as well as ones where it yields another multiple of 15; thus, the interval of equivalence, if you will, can be rather large. In order for s_e to be used, the current issue must make rather finer distinctions, and a deviation of only 5 minutes from 45 must be sufficient to put us in a different cell. With s_f, finally, the sentence can only be used to address an issue where it matters whether John's running is within 1 minute of 45 minutes. Of course, the issue is always allowed to be more precise; for example, it could always require measurement up to a precision of half a minute. The theory predicts only that the scale that is being used puts a lower bound on the fine-grainedness of the issue.

This is exactly what is needed to explain why round numerals are more frequently interpreted loosely, and admit of more deviation. For assume that the hearer does not know exactly the shape of the issue that the speaker is addressing. Then, hearing 45 (a relatively round number), she learns that at least a deviation of 15 minutes would matter for this issue, because the coarsest scale that 45 even occurs on is s_c. If, on the other hand, she hears 43 (a very non-round number), then she knows that at least differences of 1 minute must matter, because 43 only occurs on scales at least as precise as s_f. Thus, by the same reasoning as in

36 Unfortunately, it must be admitted that we cannot always get by without actually resolving the ambiguity with respect to the scale employed. While this is not necessary to determine the communicated content of a positive sentence, it cannot be avoided when computing that of a negated sentence. And since *no* is generally used in reply to a sentence that is false (i.e. whose negation is true), a hearer who disagrees with a positive sentence involving a numeral also needs to figure out the scale that is being used in order to know whether to respond with *well* or *no*. It seems to me that there is a tendency to use the most fine-grained scale that is not overprecise for the current issue.

Krifka's theory, a speaker hearing 45 is likely to assume a more coarse-grained issue than one who hears 43, and so, by interpreting the sentence with respect to that issue, to reach a more permissive interpretation.

Finally, we can treat *exactly*, or, in the case of certain expressions pertaining to time, *sharp*, completely analogously to *all*: it closes the extension gap by collapsing it with the negative extension. Thus, while (94a) is, based on a coarse scale, undefined if there is a more fine-grained measure function that maps the event of John's arrival to 3:04, (94b) is simply false in such a case.

(94) a. John came at 3 o'clock.
 b. John came exactly at 3 o'clock.

In general, a sentence like (94b) is false whenever the most precise measure function that is applicable to John's arrival yields a value other than 3 (where 3 is considered to be identical to 3:00, etc.), no matter which coarser scale is chosen — effectively, *exactly* makes the sentence behave always as if it were interpreted with respect to the most fine-grained measure function that is applicable, and so it is only usable if tiny difference matter. Again, no actual resolution of the scale-related ambiguity is necessary in order to interpret the sentence. Consequently, a sentence with *exactly* requires a precise interpretation, but of course only relative to the most precise scale that is applicable; it does not force an arbitrarily (and meaninglessly) precise measurement.

Part II

APPLICATIONS FOR A THEORY OF HOMOGENEITY

4 HOMOGENEITY AND EXHAUSTIVITY IN CLEFT SENTENCES

The idea that cleft sentences are, in their logical form, related to plural definites and that homogeneity plays an important role in the explanation of their properties was first put forward in Büring & Križ 2013. This chapter presents such a view couched in terms of the perspective on homogeneity taken in this dissertation, and extends it to capture a number of phenomena that were not considered by Büring & Križ. A comparison is made between this theory and the other most recent approach to clefts, that of Velleman et al. 2012.

4.1 EXHAUSTIVITY AND THE GAP IN CLEFTS

A cleft sentence like (1) implies that Nina invited Adam, and furthermore that she didn't invite anybody else. The latter implication is what is being referred to when clefts are said to be *exhaustive*.

(1) It was Adam that Nina invited.
$\rightsquigarrow$ Nina invited Adam.
$\rightsquigarrow$ Nina didn't invite anybody else.

It was first pointed out by Halvorsen (1978: §1.4.2), and later taken up in Horn 1981, that the exhaustivity implication of clefts is quite troublesome in that it disappears without a trace when the sentence is negated.

(2) It wasn't Adam that Nina invited.
$\rightsquigarrow$ Nina didn't invite Adam.
$\rightsquigarrow$ Nina invited somebody other than Adam.

To be sure, (2) does imply what is effectively the negation of the exhaustivity claim. That, however, is just a consequence of the existence presupposition of clefts: both (1) and (2) presuppose that Nina invited *somebody*. This presupposition, together with the meaning component that Nina didn't invite Adam explains why (2) entails that she did invite somebody else. But apart from the situations in which the existence presupposition fails, there is another class of situations in the gap between (1) and (2): those where Nina invited both Adam and somebody else. Both sentences fail to be properly true or false in such a situation.

(3) Context: *Nina invited both Adam and Miles.*
A: It was Adam that Nina invited.
B: #Yes, and / but she also invited Miles.
B: #No, she also invited Miles.
B: Well, she did invite him, but also Miles.

(4) Context: *Nina invited both Adam and Miles.*

A: It wasn't Adam that Nina invited.
B: #Yes, and she also invited Miles.
B: #No, she also invited Miles.
B: Well, she did invite him, but also Miles.

This is the puzzle of the exhaustivity inference in clefts. What is its nature, and what is its source? I will first discuss what it is not, *viz.* a quantity implicature, by way of which I say nothing new (section 4.2). I will then first give a short presentation of Velleman et al.'s (2012) theory (section 4.3), and proceed to give an implementation of Büring & Križ's (2013) idea that the logical form of a cleft involves a definite description and that the gap can be explained as being due to homogeneity (section 4.4). These two theories will then be compared with respect to how they deal with a number of further observations regarding cleft sentences (sections 4.5 and 4.6). A conclusion naturally follows (section 4.7).

4.2 Really not a quantity implicature

Disappearance under negation is one of the prominent properties of quantity implicatures. This had led Horn (1981) to argue that the exhaustivity implication in clefts is such a conversational implicature, and approach which, as has been pointed out by Velleman et al. (2012) and Büring & Križ (2013), is quite hopeless.

Regular, i. e. non-cleft, sentences already carry an exhaustivity implicature when used in the contexts in which a cleft is appropriate.

(5) A: Who did Nina invite?
B: She invited Adam. $\rightsquigarrow$ She invited only Adam.

This implicature is straight-fowardly derivable in the usual way of all quantity implicatures: if something stronger were the case, the speaker should have said so. But it is also quite readily cancellable:

(6) A: Who did Nina invite?
B: She invited Adam. And she also invited Ginger.

This is what sets clefts apart: their exhaustivity implication is very much not cancellable.

(7) A: Who did Nina invite?
B: It was Adam she invited. #And she also invited Ginger.

There is a further reason why exhaustivity in clefts cannot be a quantity implicature — even one that somehow manages to be obligatory. If it were, then one can imagine the positive cleft in (3) being judged as neither really true nor really false in a situation where its literal meaning — that Nina invited Adam — is true but the exhaustivity implicature is false.[1] However, the negative cleft in (4) should just be plainly false in that situation — there is no implicature there, and so no

1 The experiments in Križ & Chemla 2015 show a picture along these lines for the familiar scalar item *some*.

reason why it should be judged as lacking a truth value.[2] The fact that it is, in fact, neither true nor false is incompatible with any sort of implicature theory.

4.3 THE INQUIRY TERMINATION THEORY

Velleman et al. (2012) (VEA) present a theory according to which the meaning of a cleft sentence is, effectively, the result of applying a special focus-sensitive operator to the non-cleft version of the sentence with the same focus structure. To see how it works, it is best to start with a quick look at how the exclusive focus-sensitive particle *only* is traditionally analysed. It is standard since the seminal work of Rooth (1992) to assume that *only* has a meaning along the following lines: it makes the prejacent[3] into a presupposition, while its assertion is the negation of all the relevant logically stronger focus alternatives of the prejacent. Take, for example, (8a). The prejacent (8b) is presupposed, and assuming that the individuals under discussion are Adam, Miles, and Agatha, the focus alternatives are those in (8c).[4]

(8) a. Nina only invited ADAM.
b. Nina invited Adam.
c. {Nina invited Adam, Nina invited Miles, Nina invited Adam and Miles, Nina invited Adam and Agatha, ...}

Of those alternatives, three entail the prejacent:

(9) a. Nina invited Adam and Miles.
b. Nina invited Adam and Agatha.
c. Nina invited Adam, Agatha, and Miles.

The assertive component of (8a) is then that these three alternatives are false, i. e. the proposition in (10a). Together with the presupposed prejacent, this entails (10b).

(10) a. Either Nina didn't invite Adam, or she invited nobody other than Adam.
b. Nina invited nobody other than Adam.

In their analysis of clefts, VEA use the same building blocks that are found in the meaning of *only*: the only difference is that assertion and presupposition are reversed.

(11) It was ADAM that Nina invited.

2 There is no evidence that speakers do something so absurd as to judge a sentence as lacking a definite truth-value in a situation where it is not true, but its negation has an implicature that is true. Indeed the aforementioned experimental results confirm that they do not.

3 That is, the corresponding sentence without *only*.

4 In fact, VEA speak in terms not of focus alternatives, but of alternative answers to the question under discussion. The role of focus is only to constrain what the question under discussion that can be addressed with the sentence is. Since the difference is immaterial for current purposes, I present the simplified picture.

ASS Nina invited Adam.

PRES All stronger focus alternatives are false, i. e. either Nina didn't invite Adam, or she invited nobody other than Adam.

In the case of an unnegated cleft, the conjunction of assertion and presupposition entails that Nina didn't invite anybody else. Once the cleft is negated, however, the presupposition adds no new entailments: the assertion — that Nina didn't invite Adam — is just its first disjunct, and so the presupposition is automatically true when the assertion is.

(12) It wasn't ADAM that Nina invited.

ASS Nina didn't invite Adam.

PRES Either Nina didn't invite Adam, or she invited nobody other than Adam.

The existence presupposition of clefts has been omitted in the above presentation, and indeed it is not, according to VEA, a matter of the meaning of clefts, but rather a pragmatic epiphenomenon. Essentially, they assume a picture in which the rules of information structure entail that a sentence like (12) can only be used when the question under discussion is the corresponding *wh*-question:

(13) Who did Nina invite?

This question has an existential presupposition. When it is not clear that Nina invited anybody at all, (13) cannot be the question under discussion.[5]

4.4 CLEFTS AS IDENTITY STATEMENTS

Büring & Križ (2013) take a very different approach. Following Percus 1997, they assume that a cleft's logical form is that of a copula sentence with a definite description. Thus, (14a) means essentially the same thing as (14b), with the caveat that the definite description in the cleft must be regarded as number-neutral and doesn't contain a restrictor noun like *people*.

(14) a. It was Adam that Nina invited.
b. The person who Nina invited was Adam.

They argue that if identity, like other binary relations between (pluralities of) individuals, shows homogeneity effects, this explains what we observe as exhaustivity in clefts.

4.4.1 *Homogeneous Identity*

Identity, as we are used to thinking about it mathematically, is an all-or-nothing matter: two objects are either identical, or they are non-identical. Considerations

5 The existential presupposition of *wh*-questions is a relatively standard assumption, although of course it faces the challenge of the answer *nobody*. VEA argue that this is, in fact, not really an answer of the question, but a way of rejecting the question altogether.

of whether they overlap, or one is contained in the other, don't enter into the picture, and so the statement that an entity is identical to a mereological sum that properly contains it (e.g. $a = a \oplus b$ when $a \neq b$) is plainly false and its negation ($a \neq a \oplus b$) is true. However, identity statements in natural language do not behave the same way. In a scenario where everybody is guilty, (15a) is surely not true. But neither is it accurate to assert its negation (15b), even though the sum of all girls is clearly not identical to the sum of all the guilty people. What (15b) seems to say is something stronger: that the girls aren't even *among* the culprits. Both sentences, being neither true nor false, are naturally rejected with the *well*-variant of (15c).

(15) Context: *All the young people together are guilty of some transgression.*
   a. The culprits are the girls.
   b. The culprits aren't the girls.
   c. Well / #Yes / #No, ALL the young people are guilty.

This, of course, is exactly what we expect if identity is subject to the homogeneity constraint in natural language. If this is so, then "A is B" means that A is identical to B, whereas "A isn't B" means that A and B do not overlap. Both sentences are neither true nor false when A and B are not identical, but do overlap.

The example in (15) only shows that "A is B" is undefined if A is a proper part of B. But the same holds in the reverse situation: when B is a proper part of A. (16a) is clearly not true in a situation where only the girls are actually guilty, but neither is (16b).

(16) Context: *Only the girls are guilty.*
   a. The culprits are the young people.
   b. The culprits aren't the young people.
   c. Well / #Yes / #No, (but) SOME of them.

Indeed, plain overlap gives rise to the same judgments: neither (17a) nor (17b) are true of a situation where the guilty parties are a proper subset of the girls.

(17) Context: *Some of the girls, and some of the lads are guilty.*
   a. The culprits are the girls.
   b. The culprits aren't the girls.
   c. Well / #Yes / #No, you can't say that. Some of them and some of the lads are guilty.

Thus I take natural language identity statements to employ a homogeneous identity relation as defined below, which is just another instance of a homogeneous relation closed under pointwise fusion (cf. also section 2.4.1).

DEFINITION 4.1. (Homogeneous Identity)

$$[\![a \doteq b]\!] = \begin{cases} 1 & \text{iff } [\![a]\!] = [\![b]\!] \\ 0 & \text{iff } \neg\exists x : x \preceq [\![a]\!] \wedge x \preceq [\![b]\!] \\ \# & \text{otherwise} \end{cases}$$

4.4.2 *Explaining Exhaustivity*

If clefts are identity statements with definite descriptions, then not only does the existence presupposition follow immediately — since definite descriptions presuppose that their restrictor is non-empty — , but so does the exhaustivity implication. Since the logical form of (18a) is (18b), it is only true if the sum of all people who came is identical to Adam, i. e. if Adam came and nobody else did.

(18) a. It's Adam who came.
b. $(\iota x.came(x)) \doteq Adam$

At the same time, its negation (19a), with the logical form (19b), is true if Adam doesn't *overlap* with the sum of all people who came, that is, if Adam didn't come.

(19) a. It isn't Adam who came.
b. $(\iota x.came(x)) \not\doteq Adam$

In a situation where Adam and Miles came, then, both of these sentences are neither true nor false, but suffer a homogeneity violation. This is exactly the basic pattern that was in need of explanation.

Furthermore, we predict that in the situation in (20), (20a) is also neither true nor false. $\iota x.came(x)$ evaluates to Nina, but $\iota x.girl(x)$ (in this context) denotes the sum of Nina and Agatha. These two sums are not identical, but one is contained in the other, and so the homogeneous identity statement is neither true nor false.

(20) Context: *Of Nina, Agatha and Miles, only Nina came.*
a. It was the girls who came.
b. $(\iota x.came(x)) \doteq (\iota x.girl(x))$

Similarly, a homogeneity violation and consequent undefinedness is predicted in a case of mere overlap without containment, since the homogeneous identity relation is neither true nor false of the sum of Agatha and Miles ($\iota x.came(x)$)on the one hand, and the sum of Nina and Agatha ($\iota x.girl(x)$) on the other, since these sums are not identical, but have a part in common.

(21) Context: *Agatha and Miles came, but Nina didn't.*
a. It was the girls who came.
b. $(\iota x.came(x)) \doteq (\iota x.girl(x))$

While the judgements may not be entirely clear, the predictions seem reasonable. Empirical confirmation or disconfirmation might be achievable by an application of Križ & Chemla's (2015) paradigm to cleft sentences.[6]

Note that VEA also don't predict plain falsity in these two cases once the regular homogeneity of distributive predicates is taken into account. According

6 It should be noted that on the above two points, the present theory's predictions diverge from those of Büring & Križ 2013. The implementation there amounts effectively to asymmetrically homogeneous identity:

to their theory, the assertive component of (20a) is (22). When only one of the girls came while the other didn't, that is neither true nor false.

(22) The girls came.

The difference is just that on their theory, it is not the falsity of the presupposition of clefts that causes the undefinedness in this case, but an independent violation of homogeneity in the assertive component. The same reasoning applies to (21a).

4.4.3 *Non-Maximal Clefts*

I have argued in chapter 3 that homogeneity and the potential for loose usage, more technically referred to as non-maximality, are correlated. Since clefts are a homogeneous construction, they have the potential for non-maximal readings, and if such could be found, it would constitute confirmation of the theory presented.

In particular, the theory predicts that clefts can be used non-exhaustively when the presence of additional individuals that fulfill the predicate is irrelevant for the purposes of the conversation. Such situations seem to be quite rare, but an example from section 3.3.3 can be felicitously turned into a cleft.

(23) Context: *Little Sandy built a model plane together with her father.*
A: Who built that wonderful model plane?
B: It was Sandy who built it!

The purpose of the conversation is to establish who gets the credit for the creation of the model plane. In a situation such as this, it is perfectly plausible for the father to give all the credit to the child. Thus, for current purposes, his participation doesn't matter. This strikes me as a nice explanation for the example which has the benefit that it does not require one to postulate that the lexical predicate *build* is simply reinterpreted.

4.5 THE ROLE OF FOCUS

So far, the analysis presented has completely disregarded the well-known fact that clefts involve prosodic focus; in particular, the pivot of the cleft must always either be focussed as a whole or contain a focussed element. The semantics in terms of identity statements, which makes no mention of alternatives of any kind, which are usually invoked when focus plays a role, doesn't seem to have a way of integrating the prosodic information. However, VEA rightly point out that ignoring focus information leads one to predict that (24) means that Mr. Brown's

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DEFINITION 4.2. (Asymmetrically Homogeneous Identity)

$$[\![a \doteq b]\!] = \begin{cases} 1 & \text{iff } [\![a]\!] = [\![b]\!] \\ 0 & \text{iff } \neg[\![b]\!] \preceq [\![a]\!] \\ \# & \text{otherwise, i.e. iff } [\![b]\!] \prec [\![a]\!] \end{cases}$$

This leads to the prediction that both (20a) and (21a) are plainly false in the contexts given.

eldest daughter was the only person at the party, which it clearly doesn't. Rather, what the sentence conveys is that Mr. Brown's eldest daughter was the only daughter of Mr. Brown's who was at the party. The exhaustivity implication is effectively restricted to the set of Mr. Brown's daughters.

(24) It was Mr. Brown's ELDEST daughter who attended the party.

4.5.1 *Kinds of Focus Sensitivity*

Beaver & Clark (2003, 2008) distinguish two different ways in which meaning can depend on focus: *conventional focus-sensitivity* and *pragmatic focus-sensitivity*. Conventional focus-sensitivity is seen in a small number of items such as *only*, *even*, and *also*, which make use of focus-related information in their lexical semantics. On the other hand, there is a large number of operators whose interaction with focus is mediated by pragmatic mechanisms and not part of their lexical semantics, including negation and quantificational adverbs like *always*.

VEA argue that clefts fall into the category of conventionally focus-sensitive constructions based on a diagnostic presented by Beaver & Clark. Conventionally focus-sensitive elements need a prosodic focus in their scope and cannot associate with a reduced pronominal form, whereas pragmatically focus-sensitive elements do not have this requirement. This is illustrated in (25) (from Velleman et al. 2012: 448).

(25) You can see John, but can you see Mary?
   a. Actually, I can ONLY see MARY / see HER / #see 'er.
   b. Yes — I can ALSO see MARY / see HER / #see 'er.
   c. No — I CAN'T see MARY / see HER / see 'er.
   d. Yes — I can ALWAYS see MARY / see HER / see 'er.

Clefts, VEA point out, pattern with conventionally focus-sensitive operators here: the pivot must contain a prosodic focus, and it is impossible for it to be a reduced pronominal.

(26) Actually, it's MARY / it's HER / #it's 'er who I CAN see.

4.5.2 *Definite Descriptions and Pragmatic Focus Sensitivity*

To see how the facts related to focus may be accounted for in the definite description theory of clefts, it is useful to first have a look at definite plurals. It turns out that, in a given context, (24) can be replaced by a sentence with an overt definite description and the same focus with essentially no change in meaning. Just as the exhaustivity implication of the cleft would be restricted to the set of Mr. Brown's daughters, so is the definite description in (27): B's response does not mean that Mr. Brown's eldest daughter was the only young female guest at the party.[7]

(27) A: Which one of Mr. Brown's daughters came to the party?
B: The girl who came was Mr. Brown's ELDEST daughter.

I suggest that what happens here is that the domain of the definite description is restricted in such a way as to ensure that the resulting proposition is a relevant answer to the question, and the only girls who are relevant to A's question are Mr. Brown's daughters. But now of course the very same thing can explain the meaning of (24). Given its information structure, the sentence can only be uttered to answer a question like A's in (27); and given that it is used to address such a question, the domain of the definite description in its logical form has to be chosen in such a way as to make it relevant, and so in this particular case, it will be restricted to the set of Mr. Brown's daughters.

What, then, of VEA's argument for the conventional focus-sensitivity of clefts? It turns out that, for whatever reason, reduced pronouns do not appear in copula constructions, either.

(28) a. #I'm telling you the girl I saw WAS 'er.
b. #The girl I saw WAS 'er.

Thus, while I have no explanation for why reduced forms are impossible in clefts and copula sentences, the parallel is not broken.

Furthermore, there is another diagnostic from Beaver & Clark 2008 with respect to which clefts differ from conventionally focus-sensitive constructions and actually pattern with the pragmatically focus-sensitive ones: *only* and its ilk cannot associate with fronted *wh*-words, while *always*, for example, can.

(29) A: #Who did Adam only call?
B: Adam only called NINA.

(30) A: Who does Adam always call?
B: Adam always calls NINA.

The pivot of a cleft, too, can be a moved *wh*-word:

(31) a. Who was it that Adam called?
b. It was NINA that Adam called.

This indicates that there is, in fact, no conventional focus-sensitivity involved in clefts.

7 The example becomes quite unnatural when the noun *girl* is replaced by something more general like *person*, but is fine for other expressions of about the same level of specificity, such as *young lady*. It is unclear to me what principle is at play here.

4.5.3 *Clefts with Multiple Foci*

In connection with focus, there is one remaining puzzle that should not be swept under the rug. It is possible for a cleft sentence to contain *two* foci, one in the pivot and one in the prejacent. An example of such a sentence is given in (32).

(32) A: I know Adam and Nina spoke. But who called who?
B: It was ADAM who called NINA (as usual).

There is an existential presupposition that somebody called someone, and the assertion is that Adam called Nina. The exhaustivity implication, furthermore, is that nobody else called anybody. This is a challenge for the analysis defended here. Recall that focus information enters into the meaning of a cleft in the following way: prosody determines which questions the sentence can be used to address, and then the domain of the definite description in the logical form is chosen in such a way as to make the proposition expressed relevant to that question.

The logical form of B's reply in (32), according to the present theory, is (33). The problem now is that no choice of domain will change the fact that it is presupposed that somebody called Nina, because otherwise the definite description would fail to refer.

(33) $(\iota x.phoned(x, Nina)) \doteq Adam$

But while it would, of course, be most desirable to be able to explain how, exactly, (32) comes to mean what it means, it would be sufficient for the defence of the analysis presented here to show that the same puzzle arises with definite descriptions. It must be admitted that the schema applied so far for translating a cleft sentence into one with a definite description yields a somewhat odd result.

(34) A: I know Adam and Nina spoke. But who called who?
B: ?The one who called NINA was ADAM.

However, when the arguments are inverted, we suddenly have a perfectly normal utterance.

(35) A: I know Adam and Nina spoke. But who called who?
B: ADAM was the one who called NINA.

That B's answer in (35) is acceptable and, indeed, normal is, from a purely semantic point of view, just as surprising, and for the same reasons, as the existence of (32). The following explanation for the contrast between (34) and (35) was suggested by Andy Lücking (p. c.). Assume that the existence presupposition of a definite description is checked only when the complete description has been heard, and that information gained from the preceding parts of the sentence is taken into account when doing so. In B's answer in (34), the definite description is the first constituent of the sentence, so when we have heard it and check the existence presupposition, it fails. In virtue of the question asked, we know that one of Adam and Nina called the other, but it is not part of the common ground that anybody called Nina. This forces the hearer to accommodate the presupposition, which explains why the utterance is perceived as degraded. In (35), however, we have heard "Adam was the one who called" by the time we reach the end of the definite description. From this sentence fragment, we can already infer that Adam called somebody, and given the question that is being answered, we can infer that that somebody was Nina. Thus, by the time that the definite description is finished and its existence presupposition checked, it already follows from the information in the common ground that somebody (namely Adam) called Nina and so the presupposition doesn't fail.

The same explanation based on incremental sentence interpretation also applies to (32). The definite description in a cleft sentence is, basically, the relative clause, which ends at the end of the sentence. Thus, when the existence presupposition in (32) is evaluated, we have already heard "It was Adam who called", which, given the question being answered, also allows us to infer that Nina was the one who Adam called.

4.6 MODIFIED CLEFTS

4.6.1 *Clefts with Adverbs*

Clefts that are modified by non-universal adverbs of quantification usually do not constitute exhaustive answers to the questions they are used to reply to. (36) is a very clear case of this kind, where the cleft is supplemented by an additional answer.

(36) A: Who did Mary collaborate with during the last year?
B: It was usually John that she collaborated with, but sometimes she also worked with Sue.

However, there is still an element of exhaustivity here: it is understood that Mary usually collaborated with John alone. This kind of local exhaustification can be effected by *only*, too. On one reading, (37) means that in most situations, Mary collaborated with John and nobody else.

(37) Mary usually collaborated only with John.

Since the core idea of VEA's theory is to give the meaning of clefts in terms of the same building blocks as that of *only*, the same mechanism that makes (37) possible would presumably explain (36) as well.

The definite description theory of clefts, too, faces no problem with (36): it is explained in the same way as (38).

(38) A: Who did Mary collaborate with during the last year?
B: The person she collaborated with was usually John, but sometimes she also worked with Sue.

What is going on here is that the quantificational adverb *usually* takes scope over the whole sentence and binds the time or situation argument of the definite description:

(39) For most past situations *s*: the person who Mary collaborated with in *s* was John.

There are, however, also clefts with no trace of exhaustivity about them whatsoever. One such example is (40).

(40) It was, among others, the students that liked Paul.

For any analysis like VEA's in which an actual exclusion of alternatives features in the semantics of clefts, (40) poses a serious problem. It is not entirely clear what the theory would even predict, but the corresponding sentence with *only* is quite impossible.

(41) #Only the students, among others, liked Paul.

On the definite description theory, all that is required to make sense of (40) is that the predicate *be, among others, the students* is true of any plurality which properly contains the sum of all students.

Again the copula sentence version of the same sentence is possible as well.[8]

(42) a. It was, among others, the students that liked Paul.
b. Those who liked Paul were, among others, the students.

Presumably, the predicate "be, among others, the students" holds of any plurality of which the sum of all students is a proper part. Nothing more is needed from the perspective of the definite description analysis to explain these sentences.

4.6.2 *Clefts with* only

What is left is the puzzling case of clefts whose pivot contains *only*. Obviously (43a) is just as exhaustive as the *only*-free variant (43b): it is true if Adam came and nobody else did.

(43) a. It was only Adam who came.
b. It was Adam who came.

8 Examples of this construction found on the internet are, among others, the following:

(i) a. The heirs were among others his step daughter Margot and his two sons Hans Albert and Eduard.
`http://www.einstein-website.de/z_information/faq-e.html`, Dec 8th 2014
b. The founders of Helsinki's YMCA were, among others, state councillor Sakari Topelius and theology student [...] Arthur Hjelt.
`http://www.hotelarthur.fi/en/info/history/`, Dec 8th 2014
c. His teachers at the time were, among others, Feliks Rączkowski and Vladislav Oćwieja.
`https://en.wikipedia.org/wiki/Boleslaw_Ocias`, Dec 8th 2014

But the negations of these two clefts differ profoundly. Unlike (44b), which just says that John didn't come, (44a) *presupposes* that John came, and asserts that somebody else came as well.

(44) a. It wasn't only Adam who came.
b. It wasn't Adam who came.

On VEA's theory, cleft-formation keeps the assertive component of the prejacent. Thus, what (43a) asserts is that nobody who wasn't Adam came. Presumably it also keeps the presuppositions of the non-cleft version of the sentence, i. e. that Adam came. But it adds a presupposition that all stronger alternatives of the non-cleft version are false.

(45) It was only Adam who came.
ASS Nobody other than Adam came.
PRES Adam came.
PRES All stronger alternatives of "Only Adam came" are false.

It would seem that on any sensible way of spelling out this latter presupposition, it is a tautology: whether one replaces just *Adam* or the whole phrase *only Adam*, or considers any other possible answer to the immediate question under discussion, there simply cannot be a stronger alternative. Thus the additional presupposition is vacuous and it is predicted that a cleft with *only* just means exactly the same thing as a cleft without it.

This is a nice result, and it is not clear how to replicate it on the definite description theory of clefts. However, to defend the theory, it is sufficient to just perform another *reductio ad definitum* by demonstrating that copula sentences display the same phenomenon, and indeed they do.[9]

(46) a. The guests were only Adam and Miles.
ASS Nobody other than Adam and Miles came.
PRES Adam and Miles came.
b. The guests weren't only Adam and Miles.
ASS Somebody other than Adam and Miles came.
PRES Adam and Miles came.

9 A Google search yielded numerous examples of this kind, among them the following:

(i) a. You can come as infrequently as you want, so long as the weeks that you come aren't only the weeks that you have submitted.
`http://www.thehoya.com/hilltop-writers-connect/`, Dec 8th 2014
b. What makes it even scarier is that the monsters aren't only the two thugs[.]
`http://www.amazon.com/The-Incident-VHS-Victor-Arnold/dp/6301551990`, Dec 8th 2014
c. The "leaves" of this tree aren't only the leaf nodes of the original graph; they include all the nodes, as desired.
`http://www.cs.hmc.edu/~keller/courses/cs60/s98/examples/acyclic/`, Dec 8th 2014

There is, however, a potentially differentiating prediction to be found in this area when collective predicates come into the picture. Take the sentence (47a) and its non-cleft version (47b) in the stated context.

(47) Context: *A drawing room sketch was performed by Agatha and Miles.*
  a. It wasn't only Agatha who performed the sketch.
  b. Not only Agatha performed the sketch.

The inquiry termination theory, as stated in section 4.3, predicts that both of them should be equally degraded. The sentences presuppose that Agatha performed the sketch and assert that a stronger alternative of that is true as well. This only makes any sense if *perform the sketch* is forcibly reinterpreted as *perform* in *the sketch*, which we should expect to be reflected in reduced acceptability.

A definite description, however, shouldn't care whether the predicate in its restrictor is collective or distributive: it just denotes some plurality. The sentences in (48) should, therefore, be just as good as those in (49).

(48) a. It was(n't) only Agatha and Miles who performed the sketch.
  b. The performers were(n't) only Agatha and Miles.

(49) a. It was(n't) only Agatha and Miles who came.
  b. The guests were(n't) only Agatha and Miles.

It is simply not clear to me what the empirical situation is. Furthermore, it should be noted that this is not an absolute prediction as it is unknown how the copula sentences with *only* are to be analysed anyway.[10]

4.7 CONCLUSION

In this chapter, I have defended a theory of clefts which analyses them as disguised copula sentences with definite descriptions (following Büring & Križ 2013), so that (50a) has essentially the same logical form as (50b). This allows a derivation of exhaustivity effects in clefts from homogeneity, provided that identity between pluralities is a homogeneous relation.

(50) a. It was Adam and Miles that Nina invited.
  b. The people who Nina invited were Adam and Miles.

The parallels between clefts and definite descriptions that this theory predicts are, indeed, found.

I have presented two empirical arguments in favour of this approach over others. First, *contra* Velleman et al. (2012), clefts do not show all the characteristics of conventional focus-sensitivity (unlike *only*), and the import of focus on their meaning can be explained as mediated by pragmatic processes, in particular relevance-based domain restriction, that are also found with definite plurals in

10 If some type-lifting is involved and what we are dealing with is not actually a definite description of individuals but of quantifiers, for example, then information about the collectivity of the restrictor might be accessible and make a difference.

the same way (section 4.5). Second, the theory is able to account for clefts that are modified by *among others*, which pose a problem for any account that attempts to capture exhaustivity in terms of the negation of some alternatives (section 4.6.1).

5 HOMOGENEITY AND MULTIPLICITY IN BARE PLURALS

There is a question in formal semantics of whether the denotation of plural nouns like *boys* encompasses only pluralities of boys (*exclusive reading*) or also atomic individuals (*inclusive reading*). At its simplest, the puzzle is that while (1a) seems to imply that Mary saw multiple zebras, (1b) entails that she didn't see a single such animal. Thus, the plural seems to have an exclusive reading in (1a), but an inclusive reading in (1b).

(1) a. Mary saw zebras.
b. Mary didn't see zebras.

Prior engagements with the issue have resulted in analyses of the multiplicity component of the meaning of (1a) as an implicature (Sauerland 2003, Sauerland et al. 2005, Spector 2007, Zweig 2008, 2009, Ivlieva 2013). That is to say, the literal meaning of (1a) is that Mary saw one or more zebras, and there is an implicature that she didn't see only one zebra. In this chapter, I propose an alternative account of the phenomenon based on the homogeneity of plural predication and the logic presented in chapter 2.

5.1 THE IMPLICATURE APPROACH

The intuitive idea behind the implicature approach is simple: the multiplicity implicature arises because if there were only one individual, then the speaker should have used the singular form and not the plural. It is, however, not trivial to actually spell this out formally. If the plural *zebras* comprises both atomic zebras and pluralities of zebras, then (2a) and (2b) are simply equivalent: whenever Mary saw one or more zebras, there is an atomic zebra that she saw; and whenever she saw an atomic zebra, she saw one or more zebras.

(2) a. Mary saw zebras.
$\exists x : zebra(x) \wedge saw(m, x)$
b. Mary saw a zebra.
$\exists x : zebra(x) \wedge |x| = 1 \wedge saw(m, x)$

Different implicature-based theories solve this problem in different ways.

5.1.1 *Double Exhaustification (Spector 2007)*

Spector (2007) presents a theory of the multiplicity implication as a higher-order implicature: the alternative of the plural is not just the singular, but the exhaustified meaning of the singular. The singular indefinite is assumed to be a part of two lexical scales: one consists of the items $\langle a, two, three, \ldots \rangle$; the other just of the indefinite singular and the bare plural.

(3) EXH(Mary saw a zebra) = Mary saw a zebra and Mary didn't see two zebras and Mary didn't see three zebras… = Mary saw exactly one zebra.

Spector assumed that exhaustification is always applied twice. Then in order to compute the effect of the outer exhaustification in (4), one needs to look at the original sentence (4a) and its alternative (4b).

(4) EXH(EXH(Mary saw zebras)
 a. EXH(Mary saw zebras)
 b. EXH(Mary saw a zebra)

Since the bare plural has as its only alternative the singular, but the result isn't logically weaker, the exhaustification in (4a) is just vacuous and the result means that Mary saw at least one zebra. (5b), however, means that Mary saw exactly one zebra and is therefore stronger. Thus, it is negated by exhaustification.

(5) EXH(EXH(Mary saw zebras) = EXH(Mary saw zebras) and not EXH(Mary saw a zebra) = Mary saw at least one zebra and Mary didn't see exactly one zebra. = Mary saw more than one zebra.

If the sentence is negated, of course, the scalar implicature disappears because the analogues of the stronger alternatives are now weaker.

5.1.2 *Exhaustification of Event Predicates (Zweig 2008)*

Zweig's (2008) approach is quite different from Spector's and makes use of event semantics. In this framework, (6) says that there is an event such that there is a zebra x, Mary is the agent of the event, and the event is a *seeing* x-event.

(6) Mary saw a zebra.
 $\exists e : \exists x : zebra(x) \wedge |x| = 1 \wedge agent(e, m) \wedge saw(e, x)$

If the singularity requirement is omitted, the result is still equivalent.

(7) Mary saw zebras.
 $\exists e : \exists x : zebra(x) \wedge agent(e, m) \wedge saw(e, x)$

In event semantics, verb phrases are assumed to denote predicates of events, which are then existentially closed. Zweig notes that even though the plural and the singular still yield equivalent results at the sentence level, the corresponding event predicates are not equivalent.

(8) a. $\lambda e. \exists x : zebra(x) \wedge |x| = 1 \wedge agent(e, m) \wedge saw(e, x)$
 b. $\lambda e. \exists x : zebra(x) \wedge agent(e, m) \wedge saw(e, x)$

(8a) is true of an event if it is a seeing by Mary of a single zebra. (8b) is true of any even that is a seeing by Mary of any number of zebras. Now the singular version (8a) asymmetrically entails the plural version (8b), which is exactly what is needed for an implicature.

If exhaustification does not happen (only) at the sentence level, but rather at the VP level, before the event argument is existentially quantified, then the desired result is obtained, based on just the singular and the plural indefinites as alternatives of each other.[1]

(9) $\textsc{exh}((8b)) = \lambda e.(8b)(e) \wedge \neg(8a)(e) =$
$\lambda e.\exists x : zebra(x) \wedge |x| > 1 \wedge agent(e, m) \wedge saw(e, x)$

Zweig assumes Chierchia's (2004) localist theory of implicature calculation, according to which scalar implicatures are always introduced as locally as possible without weakening the overall meaning. The latter condition ensures that there is no exhaustification of the event predicate under negation, since (10a), where the event predicate is locally exhaustified, is logically weaker than (10b).

(10) a. $\neg\exists e.\exists x : zebra(x) \wedge |x| > 1 \wedge agent(e, m) \wedge saw(e, x)$
b. $\neg\exists e.\exists x : zebra(x) \wedge agent(e, m) \wedge saw(e, x)$

5.1.3 *Implicated Presuppositions (Sauerland 2003)*

According to Sauerland 2003 and Sauerland et al. 2005, the multiplicity implicature is not a scalar implicature, but an *implicated presupposition*. This concept was introduced by Heim (1991), who suggests that there is something like the maxim of quantity for presuppositions. For present purposes, it can be formulated as follows:[2]

(11) MAXIMISE PRESUPPOSITION
Among a set of felicitous alternatives, use the one with the strongest presupposition.

The standard example used to illustrate this is the indefinite article: (12a) conveys that John has more than one friend. (12b) is odd because it suggests that John has more than one mother.

(12) a. Mary saw a brother of John's.
b. #Mary saw a mother of John's.

The reason for this, Heim argues, is that singular indefinites are alternatives to singular definites. The latter, of course, have a uniqueness presupposition. If this uniqueness presupposition is fulfilled, then, by maximise presupposition, one should use the definite, so the use of an indefinite implicates that that there isn't a unique restrictor object.

(13) a. Mary saw John's brother.
b. Mary saw John's mother.

1 This presupposes that $saw(e, x)$ is only true if x is the maximal individual that e is a seeing of, not if x is just one of the individuals that were seen in e.
2 For detailed discussion, see Percus 2006, Sauerland 2008, Chemla 2008, and Schlenker 2012.

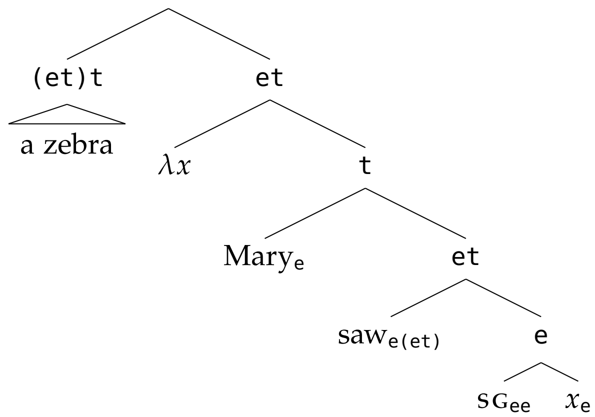
This reasoning also explains why (14a) presupposes that Mary has more than one sister even if the plural *sisters* is also true of individual sisters of Mary, and not just of pluralities of her sisters. In that case, (14a) semantically presupposes only that Mary has one or more sisters. But the alternative (14b) presupposes something stronger: that she has exactly one sister. Since the two sentences are equivalent in all situations where both of their presuppositions are fulfilled, *maximise presupposition* prescribes that a speaker should use (14b) when Mary has exactly one sister. If the speaker has instead used (14a), once can infer that Mary has more than one sister. Thus, (14a) has an implicated presupposition that Mary has more than one sister.

(14) a. Mary likes her sisters.
b. Mary likes her sister.

Unfortunately, turning this into an explanation for the multiplicity implication of bare plurals isn't as simple as all that. Sauerland (2003) and Sauerland et al. (2005) present two different attempts at this, but agree on certain assumptions: that lexical predicates like *boy(s)* include both atomic and plural individuals, and that there is a number operator of type ee at the top of every DP. These number operators map an individual to itself, but the singular operator carries a presupposition: it is defined only if its argument is an atom.[3]

(15) $[\![\textsc{sg}]\!] = \lambda x : |x| = 1.x$
$[\![\textsc{pl}]\!] = \lambda x.x$

Quantifiers are raised from under the number operator to ensure type match.[4]

(16)


It is not the singular morphology on the restrictor noun that makes (16) a quantification over atomic zebras. That morphology is uninterpreted, and the lexical predicate *zebra* is true of both atomic zebras and pluralities of them. Rather, it is the scope predicate that is only defined for individuals that are atomic.

(17) $[\![\lambda x \text{ Mary saw } \textsc{sg}\ x]\!] = \lambda x : |x| = 1.saw(m, x)$

3 These operators are supposedly the interpretation of ϕ-features. This, of course, raises the question why the morphological singular appears with mass nouns.
4 Presumably, there is a silent existential determiner that turns the noun *zebras* into a quantifier.

The precise presupposition of (16) depends on one's theory of presupposition projection, but it is certainly not stronger than that there is an atomic zebra. This leaves us with the following.

(18) a. Mary saw a zebra.
ASS $\exists x : zebra(x) \wedge saw(m, x)$
PRES $\exists x : |x| = 1 \wedge zebra(x)$
b. Mary saw zebras.
ASS $\exists x : zebra(x) \wedge saw(m, x)$
PRES None.

This is not much of a basis for deriving an implicated presupposition for (19) that yields the right meaning. Sauerland (2003) suggests that what happens is purely formal reasoning without regard for the fact that *zebra* is a distributive predicate. Then (18b) has the implicated presupposition (19a), which, together with its assertion, entails (19b).

(19) a. $\neg\exists x : |x| = 1 \wedge zebra(x)$
b. $\exists x : |x| \neq 1 \wedge zebra(x) \wedge saw(m, x)$

Only after this is the distributivity of *zebra* taken into account and the implicated presupposition, which is then revealed as a contradiction, is cancelled. The derived inference (20b), however, is somehow retained. This is rather unconvincing, and Sauerland et al. (2005) present an alternative: they assume that the principle *maximise presupposition* applies locally in the scope the existential quantifier. Then by comparison with (20a), which is Strawson-equivalent to (20b), but has a stronger presupposition, the latter gets strengthened to (20c).

(20) a. $[\![\lambda x \text{ Mary saw } \textsc{sg}\ x]\!] = \lambda x : |x| = 1.saw(m, x)$
b. $[\![\lambda x \text{ Mary saw } \textsc{pl}\ x]\!] = \lambda x.saw(m, x)$
c. $\lambda x : |x| \neq 1.saw(m, x)$

If presuppositions project at all from the scope of an existential, then (18b) has an implicated presupposition that there is more than one zebra in the world, which may be considered questionable. In any case, it is true if any only if Mary saw more than one zebra. This same reasoning, however, would also be applicable to negated sentences, and so (18a) should means that Mary didn't see more than one zebra. This forces Sauerland et al. (2005) to postulate that *maximise presupposition* applies locally only if it strengthens the meaning globally.[5]

5 Sauerland assumes that *maximise presupposition* applies locally only in the scope of existential quantifiers, not of a universal quantifier. Consequently, as he points out, (ia) is predicted to presuppose only that at least one boy has more than one sisters, not that every boy does. The superficial analogy must not lead one to think that the theory predicts an analogous reading for (ib). Like all the other theories, it predicts that (ib) is true only if every girls saw multiple zebras.

(i) a. Every boy likes his sisters.
b. Every girls saw zebras.

5.2 HOMOGENEITY: FROM DEFINITE TO BARE PLURALS

Through the lens of homogeneity, it looks like a sentence with an existential bare plural has an extension gap.

(21) Mary saw zebras.

TRUE iff Mary saw more than one zebra.
FALSE iff Mary didn't see any zebra.
UNDEFINED iff Mary saw exactly one zebra.

There is no particular, definite plurality involved here which needs to be homogeneous with respect to the predicate, but it still seems that pluralities are involved in some sense. If it were possible to derive the extension gap of (21) from the general homogeneity of plural predication, this would provide an interesting alternative view on the phenomenon, and indeed the logic I have presented in chapter 2 does have this property. For details, I refer the reader to section 2.4.4.

I will not argue that this theory of existential bare plurals is strictly superior to the implicature approach, but I will, in comparing it to variants of the latter, attempt to demonstrate that it does at least as well.

5.3 NON-MONOTONIC CONTEXTS

One point of divergence between some of the theories that have been presented is the behaviour of bare plurals in the scope of non-monotonic quantifiers. Since the pattern of projection is established, the homogeneity theory makes clear predictions for what happens when a bare plural is embedded in the scope of a non-monotonic quantifier. What I am concerned with here is the distributive reading of (22). I will, in the following, ommit the distributivity operator.

(22) Exactly 2 girls (DIST) saw zebras.

The homogeneity theory predicts the following truth conditions:

(23) Exactly two girls saw zebras.

TRUE iff two girls saw more than one zebra and all other girls saw no zebra at all.

This prediction strikes me as correct. It is shared by Spector's (2007) theory, which assumes that exhaustification negates not only stronger, but all non-weaker alternatives. In order to know the meaning of (24), we need to consider the two alternatives of (25), which are the original in (25a) and the singular alternative in (25b).

(24) EXH(EXH(Exactly 2 girls saw zebras))

(25) EXH(Exactly 2 girls saw zebras)

a. EXH(Exactly 2 girls saw zebras)
b. EXH(Exactly 2 girls saw a zebra)

Exhaustification in (25a) is vacuous.

(26) EXH(Exactly 2 girls saw zebras) = Exactly 2 girls saw at least one zebra.

In (25b), it results in the negation of all non-weaker alternatives of the original (27a).

(27) a. Exactly 2 girls saw a zebra.
b. Exactly 2 girls saw two zebras.
c. Exactly 2 girls saw three zebras.
d. ...

It is important to keep in mind that *saw two zebras* is not to be understood exhaustified, so that it is true of a girl who saw three zebras.

(28) EXH(Exactly 2 girls saw a zebra) = (27a) $\wedge \neg$(27b) $\wedge \neg$(27c) $\cdots =$ One girl saw exactly one zebra, another girl saw at least one zebra, and the rest of them didn't see any zebra.

(28) is not weaker than (26), and so it is negated by the outer exhaustivity operator.

(29) EXH(EXH(Exactly 2 girls saw zebras)) = (26) $\wedge \neg$(28) = Two girls saw more than one zebra, and no other girl saw any zebra.

Zweig, however, does not straightforwardly predict this reading. For him, there are two candidate readings, one with local exhaustification and one without.

(30) a. Exactly 2 girls saw one or more zebras.
b. Exactly 2 girls saw more than one zebra.

Since local exhaustification occurs whenever this does not lead to global weakening, the predicted reading for (23) is simply (30b). But in any case, neither of the candidates in (30) is the right meaning. The only way that the right result can be achieved is by requiring *both* readings, the one with local exhaustification and the one without, to be true.[6]

Finally, Sauerland's approach, if anything, simply predicts local exhaustification. If the local application of of *maximise presupposition* is required to strictly strengthen the global meaning, then Sauerland et al. (2005) predict no multiplicity implication at all; if it happens as long as the overall meaning isn't weakened, then (31a) should mean the same thing as (31b).

(31) a. Exactly two girls saw zebras.
b. Exactly two girls saw more than one zebra.

Thus, of the theories that have been presented, the homogeneity theory and Spector's version of the implicature approach deal with bare plurals in non-monotonic contexts without any problem, while both Zweig's and Sauerland's versions make incorrect predictions.

6 This problem is also recognised by Ivlieva (2014).

5.4 DEPENDENT PLURALS

The sentence (32) does not entail that each of my friends attends a different school, only that more than one school is attended overall. Plurals in the scope of another plural that give rise to this kind of reading are known as *dependent plurals*, a term coined by de Mey (1981).

(32) My friends attend good schools.

Note that dependent plural readings don't exist with *every*: (33) has no reading on which it means that every boy attended at least one good school, and more than one good school was attended overall.

(33) Every boy attended good schools.

Dependent plurals have been analysed as cumulative readings based on the closure of relations under pointwise fusion (Beck 2000, Zweig 2008, Champollion 2010, Ivlieva 2013). (33) is assumed to simply have the local form in (34), where $^*R$ denotes the closure of R under pointwise fusion.

(34) $\exists x : {}^*\textit{good-school}(x) \wedge {}^*\textit{attend}(\textit{my-friends}, x)$

Based on the homogeneity of relations, the homogeneity theory's predictions are straight-forward.

(35) My friends attend good schools.
TRUE iff each of my friends attends a good school and more than one school is attended overall.
FALSE iff none of my friends attends a good school.
UNDEFINED otherwise.

Various complications arise when the variants of the implicature approach are applied to this.

5.4.1 *Double exhaustification and Homogeneity in Implicatures*

On Spector's theory, the meaning of (36) depends on the two alternatives in (37).[7]

(36) $\textsc{exh}(\textsc{exh}(\exists x : {}^*\textit{good-school}(x) \wedge {}^*\textit{attend}(\textit{my-friends}, x)))$

(37) a. $\textsc{exh}(\exists x : {}^*\textit{good-school}(x) \wedge {}^*\textit{attend}(\textit{my-friends}, x))$
b. $\textsc{exh}(\exists x : {}^*\textit{good-school}(x) \wedge |x| = 1 \wedge {}^*\textit{attend}(\textit{my-friends}, x))$

The meaning of (36) is the conjunction of (37a) with the negation of (37b) (if (37b) is not entailed by (37a)). Now we have to calculate the meaning of these two. We start with (37a), which is based on the two alternatives in (38).

7 Note that dependent plurals are not discussed in the original Spector 2007. The problem about to be presented was pointed out to me by Natalia Ivlieva (p. c.), whom I thank for helpful discussion on this point.

(38) a. $\exists x : {}^*good\text{-}school(x) \wedge {}^*attend(my\text{-}friends, x)$
b. $\exists x : {}^*good\text{-}school(x) \wedge |x| = 1 \wedge {}^*attend(my\text{-}friends, x)$

(38b) says that there is one school that all of my friends attend, so it is stronger than (38a). Consequently, (38b) is negated.

(39) $\text{EXH}((37a)) = (38a) \wedge \neg(38b) = (\exists x : {}^*good\text{-}school(x) \wedge {}^*attend(my\text{-}friends, x)) \wedge \neg(\exists x : {}^*good\text{-}school(x) \wedge |x| = 1 \wedge {}^*attend(my\text{-}friends, x))$

(37b) means that all my friends attend only one and the same school. Its negation is already entailed by (39), and so it adds nothing to the meaning. (39) is the overall meaning of (36). Unfortunately, it does not only entail that more than one school is attended overall, but also that no school is attended by all of my friends. This is undesirable, as (32) doesn't actually entail this.

The whole discussion up to now has completely disregarded homogeneity. The homogeneity theory of bare plurals may not be correct, but that just means that a bare plural is also true of atoms. This does not justify ignoring the homogeneity of relations between plural individuals: the sentence in (40) is true only if no friend of mine attended any good school, and any theory needs to fall back on homogeneity to explain that. In a bivalent logic, all of them would predict that it is already false if one of my friends attended no good school.

(40) My friends attended good schools.

This raises the question of how the negation of alternatives that happens in the calculation of an implicature behaves when these alternatives have an extension gap. There are two possibilities: either the implicature is just that the alternatives are *not true*, in which case the above discussion can stand unchanged; or the alternatives are required to be *false*. Then we have to look at everything above through the lens of the logic of homogeneity. With the operator * added to the logic, the formulae that correspond to (38) are in (41).

(41) a. $\mathcal{E}({}^*good\text{-}school)(\lambda x.{}^*attend(my\text{-}friends, x))$
b. $\mathcal{E}(\lambda x.{}^*good\text{-}school(x) \wedge \mu(x) = 1)(\lambda x.{}^*attend(my\text{-}friends, x))$

(41b) is only false if none of my friends attended a good school. Because if there is one good school that some of them attended, then of that school s, the predicate $\lambda x.{}^*attend(my\text{-}friends, x)$ yields not 0, but #. In that case, the existential quantification in (41b) is, if not true, undefined, and not false. But this means that the conjunction of (41a) with the negation of (41b) is inconsistent. Consequently, (41b) cannot be negated in the course of the implicature calculation. It would mean that there is at least one good school that my friends attended (in the cumulative way), But this means that the negation of (41b) is inconsistent with (41a), and so (41b) cannot be negated in the implicature calculation. It follows that (42) corresponds to (39).

(42) $\text{EXH}(\mathcal{E}({}^*good\text{-}school)(\lambda x.{}^*attend(my\text{-}friends, x))) = \mathcal{E}({}^*good\text{-}school)(\lambda x.{}^*attend(my\text{-}friends, x))$

This just means that one or more good school was (cumulatively) attended by my friends. There is no plurality implication there yet. However, we still have to look at the correspondent of (37b) and see whether it is negated by the outer exhaustivity operator.

(43) $\text{EXH}(\mathcal{E}(\lambda x.^*\mathit{good\text{-}school}(x) \wedge \mu(x) = 1)(\lambda x.^*\mathit{attend}(\mathit{my\text{-}friends}, x)))$

The relevant alternatives are in (44).

(44) a. $\mathcal{E}(\lambda x.^*\mathit{good\text{-}school}(x) \wedge \mu(x) = 1)(\lambda x.^*\mathit{attend}(\mathit{my\text{-}friends}, x))$
b. $\mathcal{E}(\lambda x.^*\mathit{good\text{-}school}(x) \wedge \mu(x) = 2)(\lambda x.^*\mathit{attend}(\mathit{my\text{-}friends}, x))$
c. $\mathcal{E}(\lambda x.^*\mathit{good\text{-}school}(x) \wedge \mu(x) = 3)(\lambda x.^*\mathit{attend}(\mathit{my\text{-}friends}, x))$
d. $\ldots$

Conjoining the original (44a) with the negation of the other alternatives yields (45), which is true if there is exactly one good school that all my friends attended, and none of my friends attended any other good school.

(45) $$\mathcal{E}(\lambda x.^*\mathit{good\text{-}school}(x) \wedge \mu(x) = 1)(\lambda x.^*\mathit{attend}(\mathit{my\text{-}friends}, x)) \wedge \neg\mathcal{E}(\lambda x.^*\mathit{good\text{-}school}(x) \wedge \mu(x) > 1)(\lambda x.^*\mathit{attend}(\mathit{my\text{-}friends}, x))$$

This can be consistently negated by the outer exhaustivity operator and conjoined with (42), arriving at the following final meaning:

(46) $$\text{EXH}(\text{EXH}(\mathcal{E}(^*\mathit{good\text{-}school})(\lambda x.^*\mathit{attend}(\mathit{my\text{-}friends}, x)))) = (42) \wedge \neg(45) = \mathcal{E}(\lambda x.^*\mathit{good\text{-}school}(x) \wedge \mu(x) > 1)(\lambda x.^*\mathit{attend}(\mathit{my\text{-}friends}, x))$$

(46) is the desired result. Its availability hinges on the assumption that the negation of alternatives in the calculation of scalar implicatures takes into account the extension gap of those alternatives. But this is, indeed, not at all implausible. The implicature in (47) seems plausible, and it would mean that at least one boy loves *none* of his sisters. If non-truth were what counts for the purpose of implicatures, (47) would only implicate that at least one boys doesn't love all of his sisters.

(47) Some of the boys love their sisters.
$\rightsquigarrow$ Not all of the boys love their sisters.

5.4.2 *Event Exhaustification and Locality*

Let us now look at what Zweig would *prima facie* seem to predict. Exhaustification at the level of the event predicate yields the meaning in (48).

(48) $\exists e \exists x : \mathit{good\text{-}school}(x) \wedge |x| > 1 \wedge {}^*\mathit{attend}(\mathit{my\text{-}friends}(x))$

But global exhaustification is also thinkable and yields (49).

(49) $$(\exists e \exists x : \mathit{good\text{-}school}(x) \wedge {}^*\mathit{attend}(\mathit{my\text{-}friends}(x))) \wedge \neg(\exists e \exists x : \mathit{good\text{-}school}(x) \wedge |x| = 1 \wedge {}^*\mathit{attend}(\mathit{my\text{-}friends}(x)))$$

(49) is, in fact, logically stronger than (48). If the globally strongest reading were chosen, then it should be this one, but recall that according to Zweig's system, exhaustification is always as local as possible without weakening the overall meaning. Consequently, (48) is the eventual reading of (32).

5.4.3 *Implicated Presuppositions*

According to the implicated presuppositions story, we must compare the plural and the singular variants of the predicate that would be existentially quantified over, (50a) and (50b). On all individuals which fulfill the presupposition of (50b), these two predicates agree, so the application of *maximise presupposition* yields (50c).

(50) a. $\lambda x^{*}good\text{-}scool(x) \wedge .^{*}attend(my\text{-}friends, x)$
b. $\lambda x : |x| = 1.^{*}good\text{-}school(x) \wedge {}^{*}attend(my\text{-}friends, x)$
c. $\lambda x : |x| \geq 2.^{*}good\text{-}school(x) \wedge {}^{*}attent(my\text{-}friends, x)$

Existential quantification over (51c) yields to something like (51), which has the correct truth conditions.

(51) $\exists x : |x| \geq 2 \wedge {}^{*}good\text{-}school(x) \wedge {}^{*}attend(my\text{-}friends, x)$

5.4.4 *The Problem with* all

While the universal quantifier *every*, which ranges only over atoms, does not license dependent plurals, *all* does. (52) does not entail that any girl saw more than one zebra, only that more than one zebra was seen overall.

(52) All the girls saw zebras.

If definite plurals modified by *all* take part in cumulative readings normally, then there is no particular problem here and the same explanation as before can be applied. However, at least some speakers don't seem to accept cumulative readings with *all*. (53) (from Zweig 2008) seems to only have a distributive reading on which it says that ever single student read thirty paper. It is not sufficient for every student to have read at least one paper and for thirty papers to have been read by students overall.

(53) All the students read thirty papers.

But even those speakers still accept dependent plurals with *all*: (54) is quite compatible with some students having read only one paper, as long as more than one paper was read altogether.

(54) All the students read papers.

The approach that implicature theorists (Champollion 2010 and Ivlieva 2013, derived from Zweig's theory) have taken to this problem is based on the following idea. *all DP* takes as its argument an relation R between events and individuals and returns a predicate of events. It has two meaning components: one is that the relation R must hold between the event and the meaning of DP; the second component forces the relation R to be in some sense distributive with respect to the individual argument.

In Champollion's case, *all DP* has a presupposition about the relation R which it is applied to. This presupposition, which is given in (55a), amounts to the following: R fulfills this presupposition if, wheneer R holds between an event e and an individual x, then e is the sum of some events which are such that R holds between them and an atomic part of x.[8]

(55) a. $[\![\text{all DP}]\!](R)$ is defined only if
$$\forall e \forall x : R(x, e) \to ({}^*\lambda e'.\exists x' \preceq_{AT} (x) : R(x, e'))(e).$$
b. If defined, $[\![\text{all DP}]\!](R) = \lambda e.R([\![\text{DP}]\!], e)$.

Now instantiate R by the cumulative reading of *read the books*: this relation holds between an event e and an individual x if e is an event in which each part of x read some of the books and each book was read by a part of x. It is easy to see that this relation does not fulfill the presupposition of *all the students*: there is not necessarily an event in which any atomic student read (all) the books.

However, if there is a number-neutral existential in the object position, then the relation does fulfill this requirement: any event of the girls seeing zebras is a sum of events in which atomic girls see one or more zebras. Thus, *all the boys* is defined for the predicate in (56).

(56) $\lambda y.\lambda e.\exists x : {}^*zebra(x) \wedge {}^*saw(y, x, e)$

The result of the application is the event predicate in (57), to which exhaustification is then applied

(57) $\lambda e.\exists x : {}^*zebra(x) \wedge {}^*saw(the\text{-}girls, x, e)$

(58) $\textsc{exh}(\lambda e.\exists x : {}^*zebra(x) \wedge {}^*saw(the\text{-}girls, x, e))$
$= \lambda e.(\exists x : {}^*zebra(x) \wedge {}^*saw(the\text{-}girls, x, e))$
$\wedge \neg(\exists x : {}^*zebra(x) \wedge |x| = 1 \wedge {}^*saw(the\text{-}girls, x, e))$
$= \lambda e.\exists x : {}^*zebra(x) \wedge |x| \geq 2 \wedge {}^*saw(the\text{-}girls, x, e)$

A problem for this approach is that *all* is compatible with collective predicates like *meet*, which certainly do not distribute down to atoms; it is entirely meaningless to say that an atomic individual met. However, it seems plausible that a formal specification of the distributivity requirement can be found that would allow *all DP* to combine with a collective predicate as long as that collective predicate distributes down to, say, dualities. Unfortunately, *meet* doesn't actually seem to do this. (59) does not seem to imply that every single student met every single other student in the hallway. There may well have been two students at the meeting who didn't even see each other because they were on opposite sides of the crowed.

8 This is not exactly the formulation given by Champollion himself, but it amounts to the same thing.

(59) The students met in the hallway.

Ivlieva's (2013) theory of *all* avoids this problem by writing the distributivity component into the asserted meaning of *all*:

(60) $[\![\text{all DP}]\!] = \lambda R.R([\![\text{DP}]\!], e) \wedge \forall x \preceq_{AT} [\![\text{DP}]\!] : \exists e' \preceq e : R(x, e')$

Applied to *saw zebras*, *all the girls* yields (61), which can be exhaustified to obtain the desired meaning.

(61) $\lambda e.\exists x : {}^*zebra(x) \wedge {}^*saw(the\text{-}girls, x, e) \wedge \forall y \preceq_{AT} the\text{-}girls : \exists e' \preceq e : \exists x' : {}^*zebra(x') \wedge {}^*saw(y, x', e')$

(62) $\text{EXH}((61))$
$= \lambda e.(\exists x : {}^*zebra(x) \wedge {}^*saw(the\text{-}girls, x, e) \wedge$
$\forall y \preceq_{AT} the\text{-}girls : \exists e' \preceq e : \exists x' : {}^*zebra(x') \wedge {}^*saw(y, x', e'))$
$\wedge \neg(\exists x : {}^*zebra(x) \wedge |x| = 1 \wedge {}^*saw(the\text{-}girls, x, e) \wedge$
$\forall y \preceq_{AT} the\text{-}girls : \exists e' \preceq e : \exists x' : {}^*zebra(x') \wedge |x'| = 1 \wedge {}^*saw(y, x', e'))$
$= \lambda e.\exists x : {}^*zebra(x) \wedge |x| \geq 2 \wedge {}^*saw(the\text{-}girls, x, e) \wedge$
$\forall y \preceq_{AT} the\text{-}girls : \exists e' \preceq e : \exists x' : {}^*zebra(x') \wedge {}^*saw(y, x', e')$

This shares with Champollion's approach the problem that it is not compatible with Zweig's assumption that exhaustification is as local as possible (which in turn is needed to deal with dependent plurals with a definite plural in subject position): exhaustification below *all* does not yield the desired dependent plural reading, but rather a distributive reading where each girl saw several zebras. Ivlieva therefore gives up this requirement and assumes that all exhaustification sites are available as long as they don't weaken the overall meaning. The undesirable reading discussed in section 5.4.2 above, which results from higher exhaustification, then has to be assumed to exist in principle, but be difficult to observe because the other reading tends to be preferred for some reason (one may speculate that perhaps it is easier to process and tends to be more relevant).

Ivlieva's meaning for *all* should also be adapted to distribute not strictly to atoms: one would again need to have something like "distributivity as far as possible, given the nature of the predicate", e. g. to dualities in the case of *meet*. But then it would straightforwardly follow that *all DP* is compatible with *meet*: the predicate just has to hold of all the constituent dualities *in this particular case*, and not always, as Champollion's theory would require. The prediction is then that while (59) doesn't require every student to have met every other student, (63) does. Whether this is correct is intuitively unclear to me.

(63) All the students met in the hallway.

It should also noted that Ivlieva's lexical entry for all predicts a strange reading when a numeral quantifier takes scope below *all* without an intervening distributivity operator. Take the relation predicate in (64).

(64) *see two zebras*
$\lambda x.\lambda e.\exists y : |y| = 2 \wedge {}^*zebra(y) \wedge {}^*see(x, y, e)$

Application of *all the girls* to this yields (65).

(65) $\lambda e.\exists y : |y| = 2 \wedge {}^*zebra(y) \wedge {}^*see(the\text{-}girls, y, e) \wedge \forall x \preceq_{AT} the\text{-}girls : \exists e' \preceq e : \exists y' : |y'| = 2 \wedge {}^*zebra(y') \wedge {}^*see(the\text{-}girls, y', e')$

This is something rather curious: it is intermediate in logical strength between the distributive reading, which just requires that every girl saw two zebras, and the reading on which the numeral has wide scope, which requires that every girl saw the same two zebras. (65) means that every girl saw two zebras and that in addition there are two zebras such that each of them was seen by at least one girl and every girl saw at least one of them. As far as I can tell, this reading doesn't actually exist. This poses a problem for the compositional semantics: how can this reading be prevented from being generated while the regular distributive reading is available?[9]

Spector doesn't consider the problem of *all*, or of dependent plurals in general, but in principle, it can be applied to Ivlieva's lexical entry for *all*. Since the exhaustification of event predicates no longer plays a role then, the following adaptation without reference to events can be used.

(66) $[\![\text{all DP}]\!] = \lambda P.P([\![\text{DP}]\!]) \wedge \forall x \preceq_{AT} [\![\text{DP}]\!] : P(x)$

If homogeneity in alternatives were not taken into account, then, for the reasons discussed in section 5.4.1, then it would be predicted that (67) implies that there was no single zebra which was seen by all the girls, which is incorrect. (67) is quite compatible with one zebra being seen by all girls as long as at least some of the girls also saw another zebra.

(67) All the girls saw zebras.

We therefore have to phrase the analysis in terms of the homogeneous logic. In order to render the universal quantifier, assume that the constant $\mathcal{A}$ is such that $\mathcal{A}(P)(P')$ is true iff $\{x \mid [\![P]\!](x) = 1\} \subset \{x \mid [\![P']\!] = 1\}$. Then the meaning for *all* is as follows:

(68) $[\![\text{all DP}]\!] \lambda P.P([\![\text{DP}]\!]) \wedge \mathcal{A}(\lambda x.x \sqsubseteq_{AT} [\![\text{DP}]\!])(P)$

In order to compute the final meaning via double exhaustification, we have to look at the plural, the singular, and the various numeral alternatives:

(69) a. $\mathcal{E}({}^*zebra)(\lambda x.{}^*saw(the\text{-}girls, x)) \wedge \mathcal{A}(\lambda x.x \sqsubseteq_{AT} the\text{-}girls)(\lambda x.\mathcal{E}({}^*zebra)(\lambda y.{}^*saw(x, y)))$

b. $\mathcal{E}(\lambda x.{}^*zebra(x) \wedge \mu(x) = 1)(\lambda x.{}^*saw(the\text{-}girls, x)) \wedge \mathcal{A}(\lambda x.x \sqsubseteq_{AT} the\text{-}girls)(\lambda x.\mathcal{E}(\lambda x.{}^*zebra(x) \wedge \mu(x) = 1)(\lambda y.{}^*saw(x, y)))$

c. $\mathcal{E}(\lambda x.{}^*zebra(x) \wedge \mu(x) = 2)(\lambda x.{}^*saw(the\text{-}girls, x)) \wedge \mathcal{A}(\lambda x.x \sqsubseteq_{AT} the\text{-}girls)(\lambda x.\mathcal{E}(\lambda x.{}^*zebra(x) \wedge \mu(x) = 2)(\lambda y.{}^*saw(x, y)))$

d. ...

9 This problem doesn't befall Champollion's version of *all* because the relation in (64) doesn't fulfill the presupposition of *all*.

For the same reasons as discussed in section 5.4.1, single exhaustification of the plural alternative (69a) is vacuous. In order to compute the double exhaustification, we need to look at the single exhaustification of the singular alternative (69b).

First let us look at the single exhaustification of the plural version (69a). (69b), the singular alternative, is actually logically stronger than (69a), so it is negated. The exhaustification of the singular version is as follows:

(70) $\textsc{exh}((69b)) = (69b) \wedge \neg(69c) \wedge \neg \cdots =$
$\mathcal{E}(\lambda x.{}^*zebra(x) \wedge \mu(x) = 1)(\lambda x.{}^*saw(the\text{-}girls, x)) \wedge$
$\mathcal{A}(\lambda x.x \sqsubseteq_{AT} the\text{-}girls)(\lambda x.\mathcal{E}(\lambda x.{}^*zebra(x) \wedge \mu(x) = 1)(\lambda y.{}^*saw(x, y))) \wedge$
$\neg\mathcal{A}(\lambda x.x \sqsubseteq_{AT} the\text{-}girls)(\lambda x.\mathcal{E}(\lambda x.{}^*zebra(x) \wedge \mu(x) \geq 2)(\lambda y.{}^*saw(x, y)))$
$\approx$ There was one zebra that all the girls saw, and at least one girls saw only one zebra.[10]

(70) is stronger than (69a) and so is negated by the outer exhaustivity operator.

(71) $\textsc{exh}(\textsc{exh}((69a))) = (69a) \wedge \neg(70)$

(70) consists of three conjuncts, so there are three ways to make it false. The negation of the first conjunct would, due to homogeneity, mean that no girl saw any zebra and is therefore incompatible with the truth of (69a). The negation of the second conjunct would mean that one girl saw no zebra at all, which is also incompatible with (69a). Hence it must be the negation of the third conjunct that makes the negation of (70) true in (71).

(72) $\textsc{exh}(\textsc{exh}((69a))) = (69a) \wedge \neg(70) =$
$\mathcal{E}({}^*zebra)(\lambda x.{}^*saw(the\text{-}girls, x)) \wedge$
$\mathcal{A}(\lambda x.x \sqsubseteq_{AT} the\text{-}girls)(\lambda x.\mathcal{E}({}^*zebra)(\lambda y.{}^*saw(x, y))) \wedge$
$mathcal A(\lambda x.x \sqsubseteq_{AT} the\text{-}girls)(\lambda x.\mathcal{E}(\lambda x.{}^*zebra(x) \wedge \mu(x) \geq 2)(\lambda y.{}^*saw(x, y)))$
$\approx$ Every girl saw two or more zebras.

Thus, Spector's double exhaustification theory, applied to Ivlieva's meaning for *all*, yields a well-defined, but wrong result: it derives just the local plurality reading, not the dependent plural reading.

The implicated presuppositions theory doesn't fare any better. Either the existential bare plural takes scope below *all*, which yields the wrong reading: all girls will have to have seen multiple zebras. Or the existential quantifier takes scope above *all*, but it turns out that this doesn't help. The predicate that maximise presupposition is applied to will be (73a), with its singular alternative (73b). Whatever the meaning of *all*, these predicates agree on all individuals which fulfill the presupposition of (73b), and so *maximise presupposition*, applied to (73a), yields (73c).

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10 This means that the negation of (69c) is made true by the falsity of the second conjunct in (69c). The negation of the first conjunct is $\neg\mathcal{E}(\lambda x.{}^*zebra(x) \wedge \mu(x) = 2)(\lambda x.{}^*saw(the\text{-}girls, x))$, which, by homogeneity, means that at most one zebra was seen by any girl. This entails the falsity of the second conjunct: if no more than one zebra was seen by any girl, then not all girls saw more than one zebra.

(73) a. $\lambda x.{}^*zebra(x) \wedge [\![\text{all the girls}]\!](\lambda y.{}^*saw(x,y))$
b. $\lambda x : |x| = 1.{}^*zebra(x) \wedge [\![\text{all the girls}]\!](\lambda y.{}^*saw(x,y))$
c. $\lambda x : |x| \geq 2.{}^*zebra(x) \wedge [\![\text{all the girls}]\!](\lambda y.{}^*saw(x,y))$

Applying existential quantification to (73c) yields (74), which just means that there is a plurality of zebras such that all the girls saw that plurality. Unless *all the girls* can participate in cumulative readings, this will entail that all girls saw multiple zebras.

(74) $\exists x : |x| \geq 2 \wedge {}^*zebra(x) \wedge [\![\text{all the girls}]\!](\lambda y.{}^*saw(x,y))$

I regard the mystery of the word *all*—its exact meaning and selectional restrictions—as fundamentally unsolved, since both Champollion's and Ivlieva's sophisticated lexical entries still face certain problems. But no matter what the meaning of *all* is, the implicated presuppositions theory cannot explain dependent plurals under *all* as in any way based on cumulation, and neither can the homogeneity theory; unless, that is, *all* is, in fact, semantically compatible with cumulative readings after all and there is just some independent reason why distributive readings are strongly preferred with it. If cumulative readings with *all* are indeed impossible, a different analyses for these dependent plurals would be required, perhaps one of a more syntactic nature.

As a final remark, I can merely add that while I agree that many sentences with *all* in subject position do not like to receive cumulative readings, I do think that others like (75) cast some doubt on their categorical impossibility.

(75) All my friends attend these five schools.

5.4.5 *Interim Conclusion*

It has been suggested that dependent plurals are to be explained by cumulative readings. All the theories discussed are compatible with this, although Spector's requires the not implausible additional assumption that the negation of alternatives in the course of the implicature calculation takes into account the homogeneity of these alternatives. When it comes to explaining the fact that bare plurals are also possible with *all*, which doesn't normally participate in cumulative readings, only developments of Zweig's approach (Champollion 2010 and Ivlieva 2013) have anything to offer, but even those aren't entirely without problems. The exact meaning of *all* is still a puzzle that cannot be regarded as entirely understood at this point.

5.5 CONTEXT-DEPENDENCE

Experimental data from Grimm 2013 indicate that the multiplicity implication is sometimes present and sometimes absent depending on contextual factors. In one experiment, subjects were presented with a display depicting a concrete situation, and asked to answer *yes* or *no* to a question containing a bare plural, such as those in (76).

(76) a. Is this woman holding mugs?
b. Is the mug in this picture sitting next to laptops?

In the conditions where there were, in fact, multiple objects with the property in question, subjects answered *yes* 90% of the time, as expected. However, when there was only a single object, a mere 32% gave *yes*-responses.

In a second experiment, bare plurals were used in a more abstract rules-and-regulations context. Participants were presented with a question and a description of facts about a hypothetical individual, and then asked how that individual should answer the question. An example is given in (77).

(77) *Did your team terminate projects this fiscal quarter?*

EMPLOYEE FACTS: Employee's team has terminated exactly one project this fiscal quarter.

What answer should the employee give? *Yes/No*

In this experiment, subjects answered *yes* 78% of the time in singular contexts, i. e. when there was only a single instance of the bare plural noun. One sophisticated participant left an illuminating comment explaining their reasoning (reported in Grimm 2014):

> "Even though the question uses the plural of the word 'projects', the intent seems to want the employee to disclose whether any projects were terminated, not just whether more than one project was terminated."

This captures an important intuition: an inclusive interpretation arises if it is contextually irrelevant whether there is only one witness or more than one.

5.5.1 *Questions, Implicatures, and Answers*

The sense in which scalar implicatures doubtlessly disappear in questions is that the polar question p? is not interpreted as asking whether or not it is the case that p and its implicatures are true. This is shown clearly by the fact that B's answers in (78) and (79) are infelicitous.[11]

(78) Context: *Adam ate all of the apples.*

A: Did Adam eat some of the apples?
B: #No.

(79) Context: *Adam saw both Nina and Agatha.*

A: Did Adam see Nina or Agatha?
B: #No.

It strikes me as much less certain, however, that a positive answer to such a question doesn't have the associated scalar implicature.[12]

11 Unless *some* and *or*, respectively, were pronounced with focal stress in the question.
12 How, exactly, that implicature would be derived is a question that I will not explore here.

(80) A: Did Adam eat some of the apples?
B: Yes.
$\overset{?}{\rightsquigarrow}$ Adam didn't eat all of the apples.

(81) A: Did Adam see Nina or Agatha?
B: Yes.
$\overset{?}{\rightsquigarrow}$ Adam saw only one of them.

If indeed this implicature is never drawn, then Grimm's data present a serious problem for the implicature theory of the multiplicity implicature. However, this seems quite doubtful. It appears to me that whether the affirmative answers in (81) and (80) have their implicatures generally depends on the context. If the overtly asked *wh*-question is clearly all that matters, then of course there is no implicature. But if it is plausible that B thinks that A's question is just a prelude and that a more detailed inquiry is likely to follow, then the implicature is likely to be drawn.

When the question is about the completion of projects, the intuition that one subject voiced — that it just doesn't matter whether one or more projects were completed — is not lost on the implicature theory. This is just the context in which an implicature is not expected to arise, so the answer *yes* is appropriate.

When it does matter whether there was only one or more than one object, then a positive answer should have an implicature.

(82) A: Is this woman holding mugs?
B: Yes.
$\rightsquigarrow$ The woman is holding more than one mug.

This means that in a situation where the woman has only one mug, *yes* isn't an entirely appropriate answer. But this still doesn't quite explain why subjects choose the answer *no* with overwhelming majority, since (78) and (79) show that the falsity of the implicature of the declarative version of a polar question doesn't license the answer *no*. However, the following argument can be made: it must be taken into account that there was a forced choice between *yes* and *no*, and both of these answers were misleading. Subjects may simply have felt that the answer *no*, even though not literally correct, would be less misleading.

Note that in any case, Grimm's data certainly refute claims that have been made about the multiplicity implicature being obligatory (Ivlieva 2013).

5.5.2 *Non-Maximality*

The homogeneity theory has its own way of dealing with the context dependence of the multiplicity implicature. In chapter 3, I have argued that a sentence that is undefined due to a homogeneity violation can still be used if it is true enough for current purposes, where a sentence is true enough if the actual situation is, for current purposes, equivalent to one where the sentence is literally true.

This explains the answer *yes* to the question about finished projects in just the way that the subject's comment suggests. Literally, this answer is neither true nor false, but since it only matters whether any project at all was finished, it can be used nonetheless. In the question about concrete objects, apparently subjects were more inclined to think that it mattered whether there was only one or more than one, and so a larger percentage answered based on the literal meaning — of course, faced with the choice only between *yes* and *no*, they used *no* for general non-truth, that is, also in the case of undefinedness.

As far as answers to questions are concerned, this makes the same predictions as the explanation in terms of implicatures suggested above. There is, however, a differential prediction: the implicature theory predicts that (83) cannot under any circumstances be used when Mary has exactly one mug.

(83) Mary didn't have mugs.

The homogeneity theory, on the other hand, predicts that (83) can be used in such a situation, as long as having one mug is, for current purposes, equivalent to having none. This seems to be borne out: (84) doesn't strike me as at all impossible.

(84) Context: *Mary had exactly one mug.*
Mary didn't have mugs, so we couldn't drink tea together.

Unfortunately, this is not an entirely definitive argument. In the absence of contrastive focus, local exhaustification in a downward-entailing context is not a possible explanation, but it could be that *have mugs* manages to be somehow reinterpreted as *have enough mugs*. This would predict that in a context where three mugs are needed, it can still be said that Mary didn't have mugs when she had only two.

(85) Context: *Mary had only two mugs, but there were three people.*
Mary didn't have mugs, so we couldn't all drink tea together.

Only if (85) is noticeably worse than (84) does this constitute an argument for the homogeneity theory over the implicature approach. It is not intuitively clear to me what the situation is.

5.6 MASS NOUNS

Magri (2011) points out that if there is such a thing as a plurality inference for mass nouns, then the implicature approach is in trouble, since it relies on the contrast between a singular and a plural indefinite, which doesn't exist in the case of mass nouns. Magri's example of such a plurality inference is (86). (86a) would seem to require John to have more than one piece of change in his pocket, while (86b) means that he doesn't have a single piece of change.

(86) a. John has change in his pocket.
b. John doesn't have change in his pocket.

In order for the implicature theory to capture this fact, it would have to be assumed that *change* has an alternative *a piece of change*, which, in light of the fact that the latter appears quite a bit more complex, is rather dubious.

On the homogeneity theory, it would have to be assumed that *change* is undefined of single pieces of change. There is no obvious reason why this should be so, but there is also no obvious reason why it should not, and given the general deep parallels between plurals and mass nouns, it is not altogether too implausible.

5.7 conclusion

I have suggested that the meaning of plural nouns is neither inclusive nor exclusive, but something in-between: the plural noun *zebra* is neither true nor false of atomic zebras. Together with the logic for homogeneity from chapter 2, this entails that sentences with existential bare plurals are true when there is a plurality of witnesses, undefined if there is a single witness, and false if there is none at all. This explains how the multiplicity implication disappears under negation.

The theory correctly predicts the behaviour of bare plurals in embedded contexts, specifically in the scope of non-monotonic quantifiers, and is compatible with the cumulation-based explanation for dependent plurals. What it cannot explain is why dependent plurals are possible under *all*, when DPs with *all* do not participate in cumulative readings.

This approach was compared to three implicature-based theories. Spector's (2007) double-exhaustification theory yields the same results if the homogeneity of negated alternatives is taken into account. Otherwise, it still makes the correct predictions for non-monotonic contexts, but cannot deal with dependent plurals at all. Zweig's (2008) theory, which is based on exhaustification of the event-predicate, makes incorrect predictions for non-monotonic contexts, but can deal with dependent plurals and be adapted (Champollion 2010, Ivlieva 2013) to provide at least some explanation for dependent plurals with *all*, even if those are not entirely without problems. A theory based on implicated presuppositions, due to Sauerland (2003) and Sauerland et al. (2005), is conceptually the most awkward of the approaches and fares worst empirically: it can explain dependent plurals, but not with *all*, and it makes incorrect predictions for non-monotonic contexts.

When it comes to explaining how the presence of the multiplicity implication depends on contextual factors (Grimm 2013), both the implicature approach and the homogeneity theory have adequate tools at their disposal. Potential points of difference between the two are empirically too unclear to support a definitive decision.

In sum, I conclude that the homogeneity theory of the multiplicity implication is not clearly an improvement over all existing theories, but it is an interesting alternative that does as well as the best of them.

6 HOMOGENEITY AND NEG-RAISING

There is a class of intensional predicates, including *believe*, *want*, and *seem*, which as such that when they are syntactically negated, the same meaning seems to ensue as when their complement is negated. Thus, the two sentences in each of (1), (2), and (3) are basically equivalent. These predicates are known as *neg-raising* predicates (NRPs). I will occasionally refer to both (1b) and the inference from (1a) to (1b) as the *neg-raising inference*.

(1) a. Nina doesn't believe Adam will come.
  b. Nina believes Adam won't come.

(2) a. Nina doesn't want Adam to come.
  b. Nina wants Adam not to come.

(3) a. Adam doesn't seem to be here.
  b. Adam seems not to be here.

Gajewski (2005) notes a parallel between neg-raising and homogeneous plural predication and presents a theory of both in terms of presuppositions. This theory has recently been challenged by Romoli (2012, 2013), who proposes an alternative in terms of scalar implicatures. The aim of this chapter is to present and defend a theory of neg-raising couched in terms of the conception of homogeneity defended in this dissertation, on which NRPs involve homogeneous predication over pluralities of possible worlds. The syntactic view on neg-raising, recently revived by Collins & Postal (2014), will not be discussed here — for sophisticated counterarguments to such a proposal, see Romoli 2013.

6.1 TWO AND A HALF THEORIES OF NEG-RAISING

6.1.1 *The Presuppositional Theory*

Gajewski (2005) understands homogeneity as a presupposition of the distributivity operator, which he takes to be necessary for the application of a distributive predicate to a plurality. According to him, the distributivity operator is a universal quantifier with an excluded middle presupposition, i. e. it presupposes that either all or none of the individuals in its domain fulfill he predicate.

(4) The boys DIST came.

ASS $\forall x : x \preceq_{AT} \iota y.boys(y) \rightarrow came(x)$

PRES $(\forall x : x \preceq_{AT} \iota y.boys(y) \rightarrow came(x)) \vee$
$(\forall x : x \preceq_{AT} \iota y.boys(y) \rightarrow \neg came(x))$

Inspired by Bartsch 1973 and Heim 2000, Gajewski assumes that NRPs, too, are universal quantifiers that carry an additional excluded middle presupposition.

(5) Nina believes Adam will come.
ASS $\forall w : Bel(n, w) \rightarrow come(w, a))$
PRES $(\forall w : Bel(n, w) \rightarrow come(w, a))) \vee (\forall w : Bel(n, w) \rightarrow \neg come(w, a)))$

Gajewski (2007) continues to focus on NRPs themselves as soft presupposition triggers instead of exploring the connection with homogeneity any further. Romoli (2013) raises some doubts about the presuppositionality of the excluded middle statement, which partly mirror what I have discussed with respect to homogeneity in 1.7.1. He points out that the projection behaviour of the excluded middle component does not look very much like that of presuppositions in that it doesn't project from conditionals (see section 6.5.1 below) and questions: (6a) doesn't entail that Mary has any opinion on whether Bill should be hired, while (6b) is normally understood to entail that Mary used to smoke.

(6) a. Does Mary think that Bill should be hired?
b. Did Mary stop smoking?

Furthermore, just as with homogeneity violations, it is strange to protest against an utterance with an NRP on the grounds that one didn't know that the relevant excluded middle statement held, while one can do so for presuppositions:

(7) A: Does John know that Mary stopped smoking?
B: Wait a minute! I didn't know that she smoked in the first place.

(8) A: Does Mary think that John smokes?
B. #Wait a minute! I didn't know that she necessarily has an opinion about that.

These observations sow some doubt with respect to the presuppositional theory neg-raising, but I will later compare the different theories on what may be considered more substantial empirical predictions.

6.1.2 *The Scalar Implicature Theory*

Romoli (2012, 2013) presents a theory according to which scalar implicatures are behind the neg-raising inference. In particular, he derives the excluded middle statement as an implicature instead of assuming it as a presupposition. This is done by assuming that an NRP has the corresponding excluded middle predicate as a lexical alternative.

(9) a. Nina believes Adam will come.
$\texttt{bel}_n(\texttt{come}(a))$
b. Nina has an opinion as to whether Adam will come.
$\texttt{bel}_n(\texttt{come}(a)) \vee \texttt{bel}_n(\neg\texttt{come}(a))$

In the case of unnegated sentences, this is of no consequence since the scalar alternative is weaker then the original statement. Under negation, however, the entailment relationship is reversed.

(10) a. Nina doesn't believe Adam will come.
$\neg\mathtt{bel}_n(\mathtt{come}(a))$
b. Nina doesn't have an opinion as to whether Adam will come.
$\neg(\mathtt{bel}_n(\mathtt{come}(a)) \vee \mathtt{bel}_n(\neg\mathtt{come}(a)))$

In the course of exhaustification, which Romoli implements as an exhaustivity operator that negates non-weaker alternatives, following the grammaticalist tradition on scalar implicatures (Chierchia et al. 2012), the stronger alternative (10b) is negated.

(11) a. EXH(Nina doesn't believe Adam will come) =
(Nina doesn't believe Adam will come) and (Nina has an opinion on the matter) =
Nina believes Adam won't come.
b. $\text{EXH}(\neg\mathtt{bel}_n(\mathtt{come}(a))) =$
$\neg\mathtt{bel}_n(\mathtt{come}(a)) \wedge (\mathtt{bel}_n(\mathtt{come}(a)) \vee \mathtt{bel}_n(\neg\mathtt{come}(a))) =$
$\mathtt{bel}_n(\neg\mathtt{come}(a))$

The scalar implicature theory of neg-raising is faced with something of a conceptual embarrassment: the implicature calculation is based on alternatives that are, if they are even expressible at all, rather convoluted and not regularly used by anybody. This suggests that what we are dealing with here is alternatives of *meaning*, not alternative *expressions* as they are usually considered to be the basis for scalar implicatures. Furthermore, these alternatives must be lexically specified.

While such rather idiosyncratic assumptions can hardly be counted as a virtue of the theory, the most fundamental problem seems to me to be the following: how did such deeply un-Gricean implicatures ever *become* grammaticalised?

(12) a. *believe:* Nina has an opinion as to whether Adam will come.
b. *want:* ?Nina has a desire as to whether Adam should come.
c. *seem:* ???

But whatever the prior probabilities, the ultimate test for a theory is still empiricism, and so we need to compare theories on the predictions that they make for various examples.

6.1.3 *The Homogeneity Theory*

Gajewski (2005) assimilates neg-raising and the homogeneity of plural predication through an excluded middle presupposition that is shared by neg-raising verbs and the distributivity operator. I have argued in chapter 1 that this is not the right way to conceive of homogeneity, but this does not mean that Gajewski's suggestion of a deep conceptual connection between homogeneity and neg-raising must be abandoned. Neg-raising is easily assimilated to homogeneity in a way that fits with the view of the latter that I have put forward. Assume that what distinguishes NRPs is that instead of a universal quantifier, they simply involve a definite description of a plurality of worlds in their logical form — *believe*, for example, says something about the mereological sum of all worlds that are compatible with the subject's beliefs. The role of the proposition embedded under *believe*, which is a predicate of worlds, is of course to be predicated of this plurality. Thus, (13) ascribes the predicate (13a) to the world plurality (13b).

(13) Nina believes that Adam will come.
  a. $\lambda w.come_w(a)$
  b. $\iota v.Bel_w(n, v)$

The predicate in (13a) is obviously distributive — it is primitively defined for individual worlds — , and all distributive predication is homogeneous. Thus, (13) is true if Adam came in all the worlds that make up the plurality in (14b), and false only if he came in none of those worlds. Negation, as usual, just reverses truth and falsity, and so we obtain the neg-raising equivalence.

(14) Nina believes Adam will come.
  TRUE iff Adam comes in all worlds compatible with Nina's beliefs
  FALSE iff Adam comes in no world compatible with Nina's beliefs
  UNDEFINED otherwise

In fact, this approach is in principle compatible with a Gajewski-style presuppositional theory of homogeneity as well; little would change in Gajewski's theory of neg-raising in the way of predictions if he adopted a logical form with a definite description and a distributivity operator over worlds, which carries the excluded middle presupposition.

6.2 NPI LICENSING

There is a certain class of negative polarity items (NPIs), known as *strict* NPIs, which shows an interesting interaction with NRPs. They are licensed in an embedded position by matrix negation when the embedder is a NRP, but not otherwise (cf. Lakoff 1969, Horn 1978). Strict NPIs include such items as punctual *until*, *either*, and *in days/months*/etc.

(15) a. Nina didn't think that Adam would leave until tomorrow.
  b. *Nina didn't say that Adam would leave until tomorrow.

(16) a. Miles didn't think that Nina had seen Adam in days.
  b. *Miles didn't say that Nina had seen Adam in days.

(17) Context: *Adam isn't here, but Miles mistakenly thinks he is.*
  a. Adam isn't here, and Miles doesn't think Nina is here either.
  b. *Adam isn't here, and Miles didn't say Nina was here either.

In this, they differ from less demanding NPIs like the standard *any* and *ever* (*weak* NPIs), which are acceptable also under non-NRPs.

(18) Nina didn't say that Adam had ever had any money.

Gajewski (2005, 2007) points out that his approach goes together naturally with a theory of strict NPI licensing. Zwarts (1998) argues that while weak NPIs are licensed in downward-entailing contexts, strict NPIs require anti-additive contexts, which are a subset of downward-entailing contexts.

DEFINITION 6.1. (Downward-Entailingness and Anti-Additivity)

1. A function f is *downward-entailing* iff for all x, y (of the appropriate type) such that $x \subseteq y$: $f(y) \subseteq f(x)$.
2. A function f is *anti-additive* iff $f(x) \cap f(y) = f(x \cup y)$.

It is easy to see that the scope of *not say* is downward-entailing, but not anti-additive: the right-to-left direction of the required equivalence does not hold. While (19a) does entail (19b), the reverse doesn't hold. Nina may well have said that one of the two would come but she didn't know which, making (19b) true but (19a) false.

(19) a. Nina didn't say that Adam or Miles would come.
  b. Nina didn't say that Adam would come and Nina didn't say that Miles would come.

Taking into account the presupposition of *think*, however, its scope *is* an anti-additive context.

(20) a. Nina didn't think that Adam or Miles would come.
  = Nina thought that neither Adam nor Miles would come.
  b. Nina didn't think that Adam would come and Nina didn't think that Adam would come.
  = Nina thought that Adam wouldn't come and Nina thought that Miles wouldn't come.
  = Nina thought that neither Adam nor Miles would come.

That the licensing conditions for strict NPIs do, in fact, take into account presuppositions and are not checked on assertive content alone can be seen from the fact that they are not licensed in the scope of *only* (Gajewski 2007).

(21) *Only Adam arrived until 5 o'clock.

Considering only the assertive content, the scope of *only* is anti-additive, as illustrated by the mutual entailment between (23a) and (23b)

(22) Only Adam arrived.
  ASS Nobody who wasn't Adam arrived.
  PRES Adam arrived.

(23) a. Nobody who wasn't Adam arrived and nobody who wasn't Adam telephoned.
  b. Nobody who wasn't Adam arrived or telephoned.

When presuppositions are taken into account, however, this mutual entailment no longer holds. While (24a) entails that Adam both telephoned and arrived, (24b) entails only that he did at least one of the two, but not that he did both, and so the two sentence don't entail each other.

(24) a. Only Adam telephoned and only Adam arrived.
$\rightsquigarrow$ Adam telephoned and Adam arrived.
b. Only Adam arrived or telephoned.
$\not\rightsquigarrow$ Adam telephoned and Adam arrived.

Note that when taking into account presuppositions, the scope of *only* isn't even downward-entailing, and yet weak NPIs are licensed in it. This shows that weak NPIs are not sensitive to (all) presuppositions.[1]

Romoli (2013) refers to Gajewski (2011) and Chierchia (2013), who have argued that scalar implicatures are taken into account in the licensing of strict NPIs.[2] If that is so, then of course the scalar implicature theory can account for the NPIs-licensing data with NRPs

The homogeneity theory of neg-raising predicts fares no worse. It predicts, in essence, that strict NPIs should be just as natural in the scope of a definite plural that is in the scope of sentential negation.

(25) Nina didn't invite her friends until 5 o'clock.

Formally, everything is quite simple and works as in Gajewski's theory: if entailment is defined as preservation of truth in a trivalent logic, then anti-additivity, defined with that notion of entailment, is just the condition that is needed.

6.3 CANCELLATION

Under certain circumstances, it is possible for the NRPs to receive a non-neg-raising reading. This poses a challenge to the presuppositional theory because presuppositions are usually hard to ignore or accommodate locally under sentential negation.

To obtain a non-neg-raising reading, one of two special intonational patterns is required: either the negation or the NRP has to receive focal stress. It also seems to be virtually obligatory to follow up with an explicit denial of the neg-raising inference.

(26) a. Adam DOESN'T think Nina will come — he's unsure.
b. Adam doesn't THINK Nina will come — he's unsure.

This, as Gajewski (2005) points out, is reminiscent of regular presuppositions.

(27) a. Adam DIDN'T stop smoking — he never smoked in the first place.
b. Adam didn't STOP smoking — he never smoked in the first place.

1 The idea that (weak) NPIs ignore presuppositions was put forward in von Fintel 1999. For a detailed investigation of the role of presuppositions in NPI-licensing, see Homer 2011.

2 Or rather, have presented theories where this is so. I have not seen a pointed argument that shows that strict NPIs *must* be assumed to take implicatures into account. Contexts that are literally anti-additive, but cease to be when scalar implicatures are taken into account, or the other way around, seem to be difficult to find.

The second pattern, which has a distinctly metalinguistic flavour,[3] is, of course, also a familiar method of forcing local scalar implicatures.

(28) Adam didn't eat SOME of the apples — he ate ALL of them.[4]

It is much less clear that the pattern with stress on the negation exists for implicatures.[5] (29) strikes me as rather odd compared to (28).

(29) #Adam DIDN'T eat some of the apples. He ate them all.

The implicature theory, however, allows naturally for another way in which non-neg-raising readings can arise. It is a standard assumption that in the calculation of scalar implicatures, both in the Gricean and the grammaticalist tradition, only those alternatives are taken into account that are contextually relevant.[6] A simple example is the scalar implicature from *warm* to *not hot*, which, depending on the conversational context, may or may not occur. In (30a), what is relevant is whether the food needs to be heated further, and for this purpose, it doesn't matter whether it's hot or only warm, so there is no scalar implicature. When, as in (30b), the interest is in establishing whether it is *bona fide* hot, then of course there is such an implicature.

(30) a. A: Is the food warm already?
B: It's warm.
$\not\rightsquigarrow$ The food is hot.
b. A: Is the foot still hot?
B: It's warm.
$\rightsquigarrow$ The food is not hot.

Thus, in a context where it is not relevant whether Adam believes that Nina will not come or is just undecided, (31) is predicted to have a non-neg-raising reading.

(31) Adam doesn't believe Nina will come.

This predicts that it should, in principle, be possible for the neg-raising inference to be absent without special intonation, if only the context is right. Two examples from Homer forthcoming seem relevant.

(32) a. Unlike many people nowadays, my great-grandparents didn't want to spend a lot of time on the internet.
b. $\not\rightsquigarrow$ My great-grandparents wanted not to spend all their spare time on the internet.

(33) *At a job interview...*
a. I don't want to make a lot of money, you know.

3 Cf. Geurts 1998.
4 Not *that* Adam.
5 *Pace* Romoli (2013: 326f).
6 It may be noted that it has recently been argued by Romoli (2012) and Chierchia (2013) that a certain class of items have alternatives that are not subject to this relevance constraint; but never has it been claimed that this is so for scalar implicatures in general.

b. $\not\rightsquigarrow$ I want not to make a lot of money.

(32) has an additional complication: the existence presupposition from *the internet* projects through *want* so that there should be a presupposition that the speakers great-grandparents believed that the internet exists, which at the time and place of my writing this is a bizarre notion. If what happens is that this presupposition is locally accommodated below negation, then (32) doesn't show us much because the negated sentence is true just because the locally accommodated presupposition isn't. Alternatively, it could be that for whatever reason, this presupposition fails to be triggered in the first place. In that case, the irrelevance idea becomes potentially applicable. We have three different possible situations.

(34) a. The great-grandparents wanted to spend a lot of time on the internet.
b. The great-grandparents had no desire with respect to spending time on the internet.
c. The great-grandparents wanted to not spend a lot of time on the internet.

The point that the speaker wants to make is that we are not in (34a); the difference between (34b) and (34c) isn't relevant to that point. Since the indifference alternative that would be responsible for the neg-raising inference picks out exactly (34b), that alternative is not relevant: it is overinformative for current purposes.

In (33), there is no confounding presupposition. Again, we have three possibilities:

(35) a. The speaker wants to make a lot of money.
b. The speaker doesn't care about how much money they make.
c. The speaker wants not to make a lot of money.

The speaker obviously wants to assure the interviewer that they are not in situation (35a), and the difference between (35b) and (35c) is irrelevant to this point, so perhaps this is what's responsible for the absence of the neg-raising inference.

It is not clear what the presuppositional theory has to say about these cases. The homogeneity theory here agrees with the implicature theory in that contextual irrelevance is predicted to be able to weaken the neg-raising meaning, but it allows for less categorical distinctions. In chapter 3, I have argued that homogeneity enables non-maximal readings when the difference between the actual situation and one in which the sentence is strictly true is irrelevant for current purposes; that is to say, (31) is predicted to be usable whenever something is the case that is, for current purposes, equivalent to Adam believing that Nina won't come. This includes situations in which the implicature theory would also predict no neg-raising — when all that matters is whether or not he's *convinced* that she will come — as an extreme case, but it is not an all-or-nothing matter in the same way.

It would certainly be a point in favour of the homogeneity theory if non-maximality were also responsible fo the relative "weakness" of *think* and *believe* in comparison to *be convinced*. Unfortunately, this cannot be the case. The reason is that, as we saw, the fact that a sentence isn't strictly, but only non-maximally true cannot be mentioned explicitly because it must be irrelevant — this is shown in (36a) — , whereas it is possible to say (36b).

(36) a. #The professors smiled, but not all of them did.
b. I believe/think Nina will come, but I'm not quite sure.

As far as the cancellation of the neg-raising inference by focus intonation is concerned, I have observed in section 1.3.7 that stress on the homogeneity-triggering item does seem to have the ability to suppress homogeneity, provided that the item is stressable (unlike the definite article). Whether cancellation with focus on sentential negation exists is difficult to see, since such examples may also be cases of non-maximality.

In sum, the cancellation of neg-raising is an ill-understood phenomenon that does not provide a strong argument for or against any of the theories discussed.

6.4 cyclicity

An observation originally due to Fillmore (1963) is that neg-raising is *cyclic*: when one NRP is embedded under another, a negation that is syntactically at the very top is still interpreted as if it were below the *lower* NRP. This is impressively demonstrated by the example (37) (from Gajewski 2005: 52), which contains three nested NRPs.

(37) I don't imagine Bill thinks Mary wants Fred to go.
$\rightsquigarrow$ I imagine Bill thinks Mary wants Fred not to go.

It was argued by Horn (1971) that the cyclicity of neg-raising is not entirely unrestricted and depends on the identity and order of predicates. In particular, while the neg-raising behaviour can be observed in (38), it is supposed to be absent in (39).

(38) I don't think Bill wants Mary to leave.
$\rightsquigarrow$ I think Bill wants Mary not to leave.

(39) I don't want Bill to think Mary left.
$\not\rightsquigarrow$ I want Bill to think Mary didn't leave.

If this is correct, then (38) should allow for the strict NPIs *until* in the scope of the lower neg-raising verb, but (39) shouldn't. Horn points out that this prediction seems correct on the basis of the following examples.

(40) a. I don't think Bill wants Mary to leave until tomorrow.
b. *I don't want Bill to think Mary left until yesterday.

6.4.1 *Basic Cyclicity*

6.4.1.1 *The Presuppositional Theory*

On Gajewski's presuppositional theory, cyclicity is just a consequence of presupposition projection (Karttunen 1974, Heim 1992, a. m. o.). A doxastic predicate with a presuppositional complement presupposes that the predicate applies to the presupposition of that complement. When (41) is embedded under *believe* in (42), there is no presupposition that Adam has a car, but there is a presupposition that Nina believes that he does.

(41) Adam will bring his car.
PRES Adam has a car.

(42) Nina believes that Adam will bring his car.
PRES Nina believes that Adam has a car.

Now take the schematically represented sentence in (43).

(43) not [$_\alpha$ Nina believes [$_\beta$ Adam wants p)]
ASS $\neg\texttt{bel}_n(\texttt{want}_a(p))$

In virtue of the excluded middle presupposition of *want*, the node β carries the presupposition in (44). The node α carries a presupposition made up of two parts. (45a) is the excluded middle presupposition from *believe*; and (45b) arises from the projection of the presupposition of β.

(44) Presupposition of β
$\texttt{want}_a(p) \vee \texttt{want}_a(\neg p)$

(45) Presuppositions of α
a. $\texttt{bel}_n(\texttt{want}_a(p)) \vee \texttt{bel}_n(\neg\texttt{want}_a(p))$
b. $\texttt{bel}_n(\texttt{want}_a(p) \vee \texttt{want}_a(\neg p))$

The assertion $\neg\texttt{bel}_n(\texttt{want}_a(p))$ together with the presupposition (45a) entails (46a), which together with the presupposition (45b) entails (46b).

(46) a. $\texttt{bel}_n(\neg\texttt{want}_a(p))$.
b. $\texttt{bel}_n(\texttt{want}_a(\neg p))$.

6.4.1.2 *The Homogeneity Theory*

The view of homogeneity defended here, of course, also includes projection, discussed in section 1.5. A sentence with multiple neg-raising verbs is just the world-analogue of a transitive verb with two definite plural arguments, one of which contains a bound variable. In particular, (38) is analogous to (47), and homogeneity plays out accordingly.

(47) The children don't love their siblings.
TRUE iff none of the children love any of their siblings.

FALSE iff all of the children love all their siblings.
UNDEFINED otherwise.

(48) Nina doesn't believe Adam wants p.

TRUE iff none of Nina's belief worlds is such that p is the case in any of Adam's desire worlds, i.e. if Nina believes that Adam wants $\neg p$.
FALSE iff all of Nina's belief worlds are such that p is the case in all of Adam's desire worlds, i.e. if Nina believes that Adam wants p.
UNDEFINED otherwise.

In order to capture cyclicity on the view defended here, it has to be assumed that the predication of the prejacent proposition of the world-plurality provided by the neg-raising verb is mediated by a distributivity operator. If that is so, then cyclicity follows straightforwardly as well. The non-negated version of (38), (49a), has the logical form in (49b).

(49) a. Nina thinks Adam wants p.
b. $\textsc{dist}(\lambda w'.\textsc{dist}(p)(\iota v.Des_{w'}(a,v)))(\iota u.Bel_w(n,u))$

(49) is true if all Nina's belief worlds u are such that all of Adam's desire-worlds in u make the proposition p true. But it is only false if none of her belief worlds u are such that any of Adam's desire worlds in u make p true, i.e. if she believes that Adam wants $\neg p$.

To see why the distributivity operator is essential here, consider the logical form that we would obtain by β-conversion in its absence.

(50) $p(\iota v.Des_{\iota u.Bel_w(n,u)}(a,v))$

Since homogeneity does not project from restrictors (in this case, from the restrictor of the definite description of a plurality of worlds), this means that p is the case in all the worlds that are compatible with Adam's desires in *all* of Nina's belief worlds. If a world is compatible with Adam's desires only in some of Nina's belief worlds, then p doesn't have to be the case in that world. This is a very strange reading that clearly doesn't exist and isn't even easily paraphrasable, and, if it is allowed by the grammar at all, seems rather useless, which would explain why it's essentially unobservable.

6.4.1.3 *The Scalar Implicature Theory*

The scalar implicature theory, too, explains cyclicity. The alternatives relevant to (51) are obtained by replacing either *believe* or *want* or both with the disjunctive alternative, giving rise to the set in (51b).[7]

(51) Nina doesn't believe that Adam wants p.
a. $\textsc{exh}(\neg(\mathtt{bel}_n(\mathtt{want}_a(p))))$

7 Note that the alternative obtained by the simultaneous replacement of both *believe* and *want* plays no further role. This is a good thing for a grammaticalist about implicatures, as there are some doubts in the literature that such two-step alternatives are used in the calculation, for example in Fox 2007.

b. $\left\{ \begin{array}{l} \neg(\mathtt{bel}_n(\mathtt{want}_a(p))) \\ \neg(\mathtt{bel}_n(\mathtt{want}_a(p) \vee \mathtt{want}_a(\neg p))) \\ \neg(\mathtt{bel}_n(\mathtt{want}_a(p)) \vee \mathtt{bel}_n(\neg(\mathtt{want}_a(p)))) \\ \neg(\mathtt{bel}_n(\mathtt{want}_a(p) \vee \mathtt{want}_a(\neg p)) \vee \mathtt{bel}_n(\neg(\mathtt{want}_a(p) \vee \mathtt{want}_a(\neg p)))) \end{array} \right\}$

Exhaustification is done by negating those alternatives that are not entailed by the assertion, namely the middle two above, and so the result is the conjunction of the propositions in (52).

(52) a. $\neg(\mathtt{bel}_n(\mathtt{want}_a(p))])$
b. $\mathtt{bel}_n(\mathtt{want}_a(p) \vee \mathtt{want}_a(\neg p))$
c. $\mathtt{bel}_n(\mathtt{want}_a(p)) \vee \mathtt{bel}_n(\neg(\mathtt{want}_a(p)))$

(52a) and (52c) together entail (53a), and (53a) together with (52b) and the assumption of the consistency of belief[8] entails the desired (53b).

(53) a. $\mathtt{bel}_n(\neg(\mathtt{want}_a(p)))$
b. $\mathtt{bel}_n(\mathtt{want}_a(\neg p))$

6.4.2 *Partial Cyclicity*

6.4.2.1 *The Presupposition Theory*

Gajewski's presupposition-based theory straightforwardly predicts Horn's asymmetry between *believe* and *want* on account of the different presupposition projection properties of belief predicates one the one hand and desire predicates on the other. In particular, desire predicates do not have presuppositions about desires, but about beliefs: (54) clearly doesn't presuppose that Nina wants Adam to smoke or have smoked, but that she *believes* him to.

(54) Nina wants Adam to stop smoking.
PRES Nina believes that Adam smokes.

Application of the same reasoning as before now yields no cyclicity for (55).

(55) not [$_\alpha$ Nina wants [$_\beta$ Adam believe p)]
ASS $\neg\mathtt{want}_n(\mathtt{bel}_a(p))$

(56) Presupposition of β
$\mathtt{bel}_a(p) \vee \mathtt{bel}_a(\neg p)$

(57) Presuppositions of α
a. $\mathtt{want}_n(\mathtt{bel}_a(p)) \vee \mathtt{want}_n(\neg\mathtt{bel}_a(p))$
b. $\mathtt{bel}_n(\mathtt{bel}_a(p) \vee \mathtt{bel}_a(\neg p))$

Again, (57a) together with the assertion of the sentence entail (58a). But the presupposition (57b) that arises from projection is not that Nina *wants* Adam to have an opinion about p, but that she *believes* he has one (or will have one). This is not sufficient to infer (58b), and so the negation is interpreted below *want*, but

8 That is, $\neg\mathtt{bel}_n(\mathtt{want}_a(p) \wedge \neg\mathtt{want}_a(p))$.

not below *believe*: (55) means that Nina wants it not to be the case that Adam believes p; the doesn't necessarily have to believe $\neg p$.

(58) a. $\texttt{want}_n(\neg\texttt{bel}_a(p))$.
b. $\texttt{want}_n(\texttt{bel}_a(\neg p))$.

Note that on Gajewski's view, an analogous prediction is made for cases where a definite plural is embedded under *want*. Since the homogeneity of plural predication is also based on an excluded middle presupposition, that presupposition should project in the same way. Thus, Gajewski predicts that the inference in (59a) arises, but the inference in (59b) does not.

(59) a. Bill doesn't think the boys will come.
$\rightsquigarrow$ Bill thinks that none of the boys will come.
b. Bill doesn't want the boys to come.
$\rightsquigarrow$ Bill wants none of the boys to come.

This is clearly incorrect, as the inference in (59) is surely very natural. This prediction strikes me as clearly wrong: the inference in (59) is very natural. This apparent asymmetry between NRPs and definite plurals would seem to pose a challenge for the whole endeavour the conceptual unification of neg-raising and homogeneity.

6.4.2.2 *The Scalar Implicatur Theory*

First, however, we will review how the scalar implicature theory is supposed to deal with partial cyclicity. Romoli's idea is that *want* carries a presupposition and that exhaustification happens only insofar as it doesn't strengthen the presuppositions of the utterance. A reasonable way to spell out this idea formally is to say that the exhaustivity operator negates not all alternatives that are note entailed by the original sentence, but only those alternatives that are not entailed by the original sentence and don't have a stronger presupposition.[9] Furthermore, Romoli follows Heim (1992) and von Fintel (1999) in assuming that *want* presupposes that the truth of its complement is, in some sense, not yet settled in the mind of that attitude holder.

(60) Nina wants Adam to come.
PRES $\diamond_n(\texttt{come}(a)) \wedge \diamond_n(\neg\texttt{come}(a))$[10]

For (61), the two non-entailed alternatives, which would regularly be negated, are those in (62), stated together with their presuppositions.

(61) Nina doesn't want Adam to believe p.
ASS $\neg\texttt{want}_n(\texttt{bel}_a(p))$
PRES $\diamond_n(\texttt{bel}_a(p)) \wedge \diamond_n(\neg\texttt{bel}_a(p))$

9 This is a slightly streamlined version which yields equivalent results to Romoli's formulation in the cases involved.
10 $\diamond_n(p)$ means that as far as Nina is concerned, p is still possible — in the sense required by the presupposition of *want*.

(62) a. ASS $\neg\mathtt{want}_n(\mathtt{bel}_a(p) \vee \mathtt{bel}_a(\neg p))$
PRES $\diamond_n(\mathtt{bel}_a(p) \vee \mathtt{bel}_a(\neg p)) \wedge \diamond_n(\neg(\mathtt{bel}_a(p) \vee \mathtt{bel}_a(\neg p)))$, i. e. as far as Nina is concerned, it's not yet settled whether Adam is going to have an opinion on p at all.
b. ASS $\neg(\mathtt{want}_n(\mathtt{bel}_a(p)) \vee \mathtt{want}_n(\neg(\mathtt{bel}_a(p))])$
PRES $\diamond_n(\mathtt{bel}_a(p)) \wedge \diamond_n(\neg\mathtt{bel}_a(p))$

The presupposition of (62b) is just the same as that of the original assertion — that Nina doesn't know whether Adam is convinced of p or not — and so it can safely be negated. However, the presupposition of (62a) is clearly stronger than that: (62a) presupposes that Nina doesn't know whether Adam has any opinion at all. Hence (62a) cannot be negated and we end up only with the conjunction of (63a) and (63b).[11]

(63) a. $\neg(\mathtt{want}_n(\mathtt{bel}_a(p)))$
b. $\mathtt{want}_n(\mathtt{bel}_a(p)) \vee \mathtt{want}_n(\neg(\mathtt{bel}_a(p)))$

This allows inferring (64a), but not (64b), and so the same overall meaning follows as on Gajewski's theory.

(64) a. $\mathtt{want}_n(\neg(\mathtt{bel}_a(p)))$
b. $\mathtt{want}_n(\mathtt{bel}_a(\neg p))$

Unfortunately, the presupposition that has been introduced causes the scope of *not want* not to be downward-entailing anymore (let alone anti-additive) if both presuppositions and implicatures are taken into account. To be sure, the derivation of the neg-raising inference still goes through, since it adds no new presupposition.

(65) Nina doesn't want Adam to come.
ASS $\neg\mathtt{want}_n(\mathtt{come}(a))$

11 Romoli suggests an alternative phrasing in terms of Strawson-entailment: exhaustification negates all alternatives that are not Strawson-entailed by the original sentence, where Strawson-entailment is defined as below.

DEFINITION 6.2. (Strawson-entailment) f Strawson-entails g ($f \subseteq_s g$) iff
1. f and g are of type t and $f \to g$, or
2. f, g of type $\sigma\tau$ and $\forall x \in \mathrm{dom}(g) : f(x) \subseteq_s g(x)$.

This, however, is not an equivalent way of stating the procedure. If p doesn't entail q and q has a stronger presupposition than p, it is possible that p still doesn't Strawson-entail q, and thus requiring a lack of Strawson-entailment does not guarantee that presuppositions are not strengthened by exhaustification. In fact, this applies already to the example discussed in the text. (62a) is the alternative that is to remain unnegated because it has a stronger presupposition, but it is, in fact, still Strawson-entailed by the original sentence. To show this, we must find a scenario where (62a) is false and (61) is true. If (62a) is false, then the two statements in (i) hold.

(i) a. $\mathtt{want}_n(\mathtt{bel}_a(p) \vee \mathtt{bel}_a(\neg p))$, i. e. Nina wants Adam to have an opinion on p.
b. Nina is uncertain as to whether Adam has an opinion on p.

It is still quite possible, however, for (61) to be true in such a situation, namely if she wants him to believe $\neg p$.

PRES $\diamond_n(\mathtt{come}(a)) \wedge \diamond_n(\neg\mathtt{come}(a))$
IMPL $\mathtt{want}_n(\mathtt{come}(a)) \vee \mathtt{want}_n(\neg\mathtt{come}(a))$

But to see that the conjunction of these three is not downward-entailing with respect to the position of *come*, consider the sentences (66a) and (66b).

(66) a. Nina doesn't want Adam to eat vegetables.
b. Nina doesn't want Adam to eat carrots.

According to the analysis, (66a) entails that Nina thinks that it is not yet settled whether Adam is going to eat vegetables. However, neither this, nor any other component of the meaning of (66a) entails that she thinks it's not yet settled whether Adam is going to eat carrots. (66a) may well be the case while she is quite certain that he would never eat carrots.

This is a fundamental problem of the theory: any presupposition that will serve the purpose of disrupting cyclicity will also prevent downward-entailingness. The offending presupposition must be attached to the alternative that replaces $\mathtt{bel}_a(p)$ with the logically weaker $\mathtt{bel}_a(p) \vee \mathtt{bel}_a(\neg p)$, and so if the scope of *not want* were downward-entailing, then this new presupposition would also be entailed by the original $\mathtt{bel}_a(p)$ sentence, thus not hindering exhaustification.

Furthermore, Romoli's assumption about the presupposition of *want* seem justified only to the limited extent that the sentences in (67) are infelicitous.

(67) a. Nina wants Adam to seduce her tonight, but she knows he won't.
b. Nina doesn't want Adam to seduce her tonight, but she knows he will.

I conclude that the scalar implicature theory does not offer a way to explain partial cyclicity that is nearly as convincing as Gajewski's. However, as will shortly become apparent, it is far from clear that this is actually a refutation of the theory.

6.4.2.3 *The Myth of Partial Cyclicity*

It must be squarely stated that the homogeneity-based theory here has no conceptual resources for deriving a principled difference in the cyclicity of neg-raising depending on the nature and order of the verbs involved. This, however, is only a vice of the theory to the extent that Horn's claim that there is no cyclicity in (69) is, in fact, correct.

(68) I don't think Bill wants Mary to leave.
$\rightsquigarrow$ I think Bill wants Mary not to leave.

(69) I don't want Bill to think Mary left.
$\overset{??}{\rightsquigarrow}$ I want Bill to think Mary didn't leave.

It is not at all intuitively clear to me and the people I have consulted that this is indeed the case. There are, furthermore, some doubts about the argument on the basis of NPIs licensing. To be sure, (70a) is clearly bad; but (70b) has the same structure with just a slightly different lexicalisation and is perfectly acceptable. Note that (70b) has a reading which even defenders of partial cyclicity would expect to be fine, namely one where *until next week* takes scope below *want*, but above *believe*. On this reading, what the sentences says is that I have a preference about when Harry forms his belief. With the right intonational phrasing, however, the intended reading with *until next week* scoping low is clearly available as well.

(70) a. #I don't want John to believe Harry died until yesterday.
b. I don't want John to think that Harry will arrive until next week.

Numerous other examples of the form $\neg$*want*(*believe*(strong NPIs)) can be found in both English and German. Both *lift a finger* and *either* are named by Gajewski as strong NPIs and can very naturally occur in this context. (71), for example, is naturally understood as conveying that I want you to be well aware of the fact that I will not lift a finger to help you, i. e. with inner negation.

(71) I don't want you to believe that I'll lift a finger to help you.

Similarly, (72) means that Agatha wanted her friends to recognise her own difficulties at the customs — again a reading with inner negation.[12]

(72) Adam had had trouble at the customs, and Agatha didn't want her friends to think that she had got through unmolested, either.

(73) involves the German NPIs *einen blassen Schimmer (haben)* '(have) any knowledge'. When asked for assistance with something they know nothing about, a speaker might utter (73) as an honest disclosure of their ignorance.

(73) Ich will nicht, dass Du glaubst, ich hätte einen blassen Schimmer davon.
I want not that you think I had a pale shimmer of.it
'I don't want you to believe that I know anything about that.'

Thus I conclude that not only is it unnecessary for a theory of neg-raising to predict a principled asymmetry of the kind discussed, it is even undesirable. The failure of the non-presuppositional homogeneity theory to do so turns out to be a virtue over Gajewski's presuppositional version.[13] In the same vein, the fact that Romoli's explanation for the supposed phenomenon is questionable is not actually a weakness of the scalar implicature theory.

12 It is clear that *either* must have low scope under *think* here; otherwise there would be a presupposition that there is something else that Agatha doesn't want her friends to think.

13 It must be said, though, that there is no obstacle in principle to changing the presuppositional theory so as to derive full cyclicity. All that is necessary is a plausible change in how presuppositions project through *want*: in addition to (ib), (ia) should also presuppose (ic), where $\underline{p}$ is the presupposition of p.

(i) a. Nina wants p.
b. $\mathtt{bel}_n(\underline{p})$
c. $\mathtt{want}_n(\underline{p})$

This presupposition would seem inappropriate at first glance, as (iia) doesn't to presuppose (iib).

(ii) a. Nina wants Adam to bring his nephew.
b. Nina wants Adam to have a nephew.

6.5 EMBEDDED CONTEXTS

6.5.1 *Conditionals*

NRPs in the antecedent of a conditional receive their normal reading. This is clear in such examples as (74), which is naturally understood as a prohibition against flirting in case of aversion to kissing.

(74) If Nina doesn't want to be kissed, she shouldn't flirt with Adam.

(74) does not entail that Nina either wants to be kissed or is averse to being kissed; she could still just be indifferent. It also does not seem to say that Nina mustn't flirt with Adam if she's indifferent to kissing. The reading that is observed requires that whatever processes lead to the neg-raising reading of *not want* must apply locally within the antecedent of the conditional.

As Romoli points out, this is a challenge to the presuppositional theory, since presuppositions normally project from the antecedent of a conditional and are difficult to accommodate locally within it. It should be noted that such local accommodation is not entirely impossible, especially with soft triggers, which Gajewski claims NRPs to be. (75) can be uttered by someone who doesn't know whether John used to smoke, and the presupposition is understood as part of what is being conditionalised on.

(75) If John just stopped smoking, that would explain his erratic behaviour.

One may still be inclined to think that such local accommodation is more difficult to obtain with presuppositions that for neg-raising, but I cannot regard this as a conclusive argument by any means. Similarly, the possibility of local exhaustification is, of course, a staple of the grammatical view on implicatures, but it has never been denied that scalar implicatures in the antecedent of conditionals are, at the very least, comparatively rare (Howe 2011). It is not possible to draw a definitive conclusion from such intuitive quantitative differences as long as the pragmatic factors which make local exhaustification available are so ill-understood.

A difference should be mentioned that exists between definite plurals and NRPs with respect to their behaviour in conditional antecedents. As discussed in section 1.5.4, definite plurals can sometimes receive an apparently existential reading, while NRPs cannot. For example, (76a) is quite naturally understood as synonymous with (76b).

This, however, can be avoided by the addition of the ignorance presupposition Romoli suggests:

(iii) $\neg\texttt{bel}_n(p) \wedge \neg\texttt{bel}_n(\neg p)$

$\texttt{want}_n(\underline{p})$ does not entail (iii), but (iib) does entail the appropriate ignorance statement. Hence the failure of the inference from (iia) to (iib) is explained: the meaning of the latter is more than just $\texttt{want}_n(\underline{p})$, and $\texttt{want}_n(\underline{p})$ is perfectly compatible with a situation where Nina would actually prefer Adam not to have a nephew at all or doesn't care. One must not confuse the logical constant $\texttt{want}$with the English word *want*, as the first is just one component in the meaning of the latter.

(76) a. If the girls make mistakes, they will be corrected by a teacher.
b. If one of the girls makes a mistake, she will be corrected by a teacher.

(77a), however, can never mean (77b).

(77) a. If Nina thinks Adam will come, she will be very impatient.
b. If Nina thinks it possible that Adam will come, she will be very impatient.

For further remarks on this failure of the parallel between NRPs and definite plurals, see section 6.5.4 below.

6.5.2 *The Scope of Negative Existentials*

The implicature theory faces an immediate problem in that it seems to fail to predict the correct reading when the NRP is in the scope of a negated existential. Provided that presuppositions project universally from the scope of negated existentials, the presupposition theory predicts the inference from (78a) to (78b), and so does the homogeneity theory.

(78) a. Nobody thought Nina would come.
b. $\rightsquigarrow$ Everybody thought Nina wouldn't come.

The implicature theorist, however, can only negate the alternative in (79a) and, together with the original assertion, derive (79b).

(79) a. Nobody had an opinion on whether Nina would come.
b. Somebody thought Nina wouldn't come.

Romoli suggests that negative existentials are in fact decomposed for the purpose of alternative calculation, so that *nobody*, understood as *not somebody*, has an alternative *not everybody*. Allowing alternatives with replacement of multiple items, (78a) then has an alternative (80), which can be negated to obtain the right inference.

(80) Not everybody had an opinion on whether Nina would come.

The use of alternatives with multiple replacement is in itself problematic, as it has been argued within the grammaticalist tradition that implicatures are calculated only on the basis of alternatives with single replacements. Allowing alternatives with multiple replacements has side-effects such as (81).[14]

(81) Somebody thought Nina would come.
$\overset{??}{\rightsquigarrow}$ Not everybody had an opinion on the matter.

14 One might attempt to weasel out of this by saying that this inference doesn't normally arise because the alternative in (i) is irrelevant in most contexts and therefore not considered. However, it is not at all clear to me why it should be so much less relevant than (80).

(i) Everybody had an opinion on the matter.

The same reasoning used to explain (78) also predicts the inference in (82), which strikes me as very questionable and certainly not nearly as natural as that in (78).

(82) No professors failed all of his students.
$\overset{??}{\rightsquigarrow}$ Every professor failed some of his students.

Romoli (2012) recognises this, but argues based on the exploitation of Hurford's constraint that the inference does, in fact, exist. This procedure is a commonly used diagnostic tool for the possibility of implicatures and works as follows (Chierchia et al. 2012).

DEFINITION 6.3. (Hurford's Constraint)
A sentence that contains a disjunctive phrase of the form S *or* S' is infelicitous if S entails S' or S' entails S. (Hurford 1974)

(83) a. #Mary saw an animal or a dog.
b. #Mary saw a dog or an animal.[15]

Gazdar (1979) points out that there are systematic exceptions to Hurford's constraints: scalar terms, that is to say, those associated with scalar implicatures, can occur with their stronger scale-mates.[16]

(84) a. Mary solved the first problem or the second problem or both.
b. Mary read some or all of the books.

Chierchia et al. (2012) note that this can be explained if the first disjunct is locally exhaustified: in that case, Hurford's constraint is no longer violated.

(85) a. Mary solved EXH(the first problem or the second problem) or both.
b. Mary read EXH(some) or all of the books.

Thus, the reasoning goes, if it is suspected that a sentence S has a scalar implicature p, this can be tested by forming a disjunction of S with some S' that entails the literal meaning of S, but doesn't entail p. If this disjunction is acceptable, then it must be so because S is locally interpreted at $S \wedge p$, showing that p is an implicature of S.

Using this reasoning, Romoli adduces (86) in support of the supposed implicature in (82).

(86) None of my professors failed all of their students and Gennaro failed none and the others failed just some.

15 Of course, there is the acceptable (i). But one gets a clear impression that what is going on with such *or at least* is something rather special: the speaker is rethinking their previous utterance and somehow weakening their commitment.

(i) Mary saw an animal, or at least a dog.

16 There is also an ordering constraint: the semantically stronger item has to occur in the right conjunct. See Singh 2008 on this.

(i) #Mary read all or some of the books.

I must confess that in the face of such an abomination of a sentence, I remain unconvinced.

6.5.3 *The Scope of Negated Universals*

For (87a), the implicature theory predicts not only the inference in (87b) (original example from Homer forthcoming, cited by Romoli), but also that in (87c).

(87) a. Not every student wants to help me.
 b. $\rightsquigarrow$ Some student wants not to help me.
 c. $\overset{??}{\rightsquigarrow}$ Every students has a desire as to whether or not to help me.

Since (88) is a stronger alternative of (87a), it is negated by exhaustification, yielding (87c).

(88) Not every student has a desired as to whether or not to help me.

This strikes me as an inappropriately strong inference. Whether the prediction is shared by the presupposition theory depends on the precise theory of presupposition projection that is assumed. On a naive theory that predicts universal projection from universals, (88c) does follow; according to George's (2008b; 2008c; 2008a) more sophisticated theory, whose fundamental logic is similar to and was an inspiration for the logic of homogeneity presented in chapter 2, only the inference in (88b) follows. The latter is also what the homogeneity theory itself predicts.

6.5.4 *The Restrictor of Universals*

Romoli argues that a virtue of his theory is that it correctly predicts an inference that supposedly arises when a NRP is embedded in the restrictor of a universal quantifier.

(89) a. Every student who thinks I am right will support me.
 b. $\overset{??}{\rightsquigarrow}$ Some student who thinks that I am wrong will not support me.

The derivation is straightforward. (90) is a stronger alternative of (89a), and so it is negated by exhaustification. This, together with the original sentence (89a), entails (89b).

(90) Every student who had an opinion on the matter will support me.

It is far from clear to me that this inference does, in fact, exist, and Romoli seems to be aware of such doubts. The obviation of Hurford's constraint is again brought to bear as a diagnostic tool. I find (91) scarcely more convincing than (86).

(91) Either every student who thinks I am right will support me or every student who has an opinion on the matter at all will.

Note, however, that under certain plausible assumptions, the inference in (89) perhaps isn't even derived on Romoli's own terms. It is not an outrageous idea that a quantifier, in particular a universal quantifier, carries a presupposition that its restrictor is non-empty (Lappin & Reinhart 1988). Recall further from section 6.4.2 that Romoli postulated that implicatures are not allowed to add new presuppositions. Now (89a) on its own only presupposes that there is a student who thinks I'm right. (89b), however, presupposes that there is a student who thinks I'm wrong. Thus, is should not be generated as an implicature, because that would add a new presupposition.

There is a certain irony in the fact that the prohibition against new presuppositions from implicatures served the purpose of preventing cyclicity of neg-raising with *want*, which turned out to not actually be desirable. This poses something of a trilemma for the implicature theory of neg-raising: either (i) *want* doesn't have a presupposition about the belief state of the subject after all, but it plausibly does; or (ii) partial cyclicity is predicted, but it shouldn't be; or (iii) the inference in (89) is predicted, but it probably shouldn't be, either.

If presuppositions do not project from restrictors (George 2008a), then the presupposition theory does not predict the questionable inference in (89), and the same applies to the homogeneity theory. With regard to the latter, note that just as in the case of conditional antecedents, the practically existential readings that plurals sometimes receive in the restrictor of a universal have no analogue with NRPs. While (92a) is naturally understood to mean (92b), (93a) cannot be used to communicate (93b).

(92) a. Everybody who touched the statues was asked to leave.
b. Everybody who touched any of the statues was asked to leave.

(93) a. Everybody who thought Adam would touch a statue said he shouldn't be invited.
b. Everybody who thought it possible that Adam would touch a statue said he shouldn't be invited.

It might be that the pragmatic factors that are responsible for such readings with definite plurals simply never obtain with NRPs; but I have no concrete proposal to make and the observation does constitute evidence against the homogeneity theory of neg-raising.

6.6 FURTHER ISSUES

6.6.1 *High NPIs*

Gajewski points out a further issue in connection with neg-raising and NPIs, which he calls the problem of *high NPIs*. It isn't possible for sentential negation to simultaneously license an NPI in the matrix clause and a strong NPI in the scope of an NRP, while both of these are possible independently, as demonstrated by (94) (adapted from Gajewski 2005: 71).

(94) a. I didn't ever think that John will leave next week.

b. I didn't think John would leave until next week.
c. *I didn't ever think John would leave until next week.

Gajewski argues that this can be explained if presuppositions project only existentially through indefinite NPIs like *ever*, because then the environment of the strong NPI *until* in (94c) is no longer anti-additive. If presuppositions project existentially through *ever*, then (95) doesn't presuppose that I always had an opinion on whether p, but only that I sometimes did. It furthermore asserts that I was never sure of p, resulting in the overall meaning that I at least sometimes believed $\neg p$, and at all other times (if there were any such times) was uncertain about p. It doesn't entail that I believed $\neg p$ at all times.

(95) I didn't ever think that p.
PRES I sometimes had an opinion on whether p.
ASS It wasn't true at any time that I believed that p.

It is unclear to me whether this is correct, but if it is, then the scope of the NRP is indeed not an anti-additive context, since it is possible for (96a) to be true while (96b) is false: it could be that at some times, I was uncertain whether John came and whether Bill came, but certain that one of them came, making (96b) false.

(96) a. I didn't ever think that John came and I didn't ever think that Bill came.
b. I didn't ever think that John or Bill came.

This explanation is not available on the homogeneity theory, since one cannot just stipulate a different projection behaviour for *didn't ever* than for *nobody did*. The truth and falsity conditions for (97) are predicated unambiguously, making the scope of the NRP and anti-additive environment.

(97) I didn't ever think that p.
TRUE iff I always thought that $\neg p$.
FALSE iff I sometimes thought that p.
UNDEFINED otherwise.

Of course, the extent to which one can make such a stipulation for presuppositions, as Gajewski suggests, is also questionable.

The implicature theory could explain the facts if *ever* doesn't have *always* as an alternative (cf. section 6.5.2 above), in which case it would also match Gajewski's predicted truth conditions for (95).

6.6.2 *Factive NRPs*

Furthermore, Gajewski argues that his approach predicts the old observation by Kiparsky & Kiparsky (1970) that there are no factive NRPs. For example, there is no verb *know** which is just like *know* except that the inference from (98a) to (98b) is valid.

(98) a. John doesn't know* that p.

b. John know* that $\neg p$.

By the factivity of *know** and the projection of presuppositions through negation, (98a) presupposes p, while (98b) presupposes $\neg p$. Two sentences can obviously not stand in an entailment relationship when they have contradictory presuppositions, and so any non-syntactic theory of the neg-raising inference is able to explain this fact.

But there is also another type of factive predicates which doesn't exist. One can imagine a predicate *know***, which just like *know* presupposes its complement, but has an assertive component that shows neg-raising behaviour: it is only false if the subject believes the negation of the complement, whereas *know* is also false if the subject is uncertain (as long as the complement is true).

(99) John doesn't know** that p.

TRUE iff p and John believes p.
FALSE iff p and John believes $\neg p$.
UNDEFINED otherwise.

Predicates like *know*** do not seem to be found in natural languages. Gajewski's presuppositional theory doesn't explain this fact, since *know*** could be derived from *know* by simply adding a presupposition that the subject has an opinion about the truth of the complement. Similarly, all the homogeneity theory would require is the replacement of the universal quantifier in the assertive component of *know* with a definite description. The implicature theory also doesn't immediately preclude the existence of *know***: it would merely have to have *has an opinion as to whether* as an alternative, just like *believe* does, although perhaps a more sophisticated theory of alternatives and about the interaction of implicatures and presuppositions might eventually yield a more principled explanation.

6.7 CONCLUSION

In this chapter, I have discussed and compared in terms of their predictions three theories of neg-raising: Gajewski's (2005) presuppositional theory, Romoli's (2013) scalar implicature theory, and the homogeneity-based theory, which, inspired by Gajewski's observations, assumes that neg-raising verbs do not contain universal quantifiers over worlds, but rather distributive (and hence homogeneous) predication over pluralities of worlds. I judge the comparison to be ultimately inconclusive.

There is some evidence against all of these theories. The implicature theory is conceptually awkward, postulating alternatives that are, in the worst case, simply unpronounceable, and it predicts questionable inferences for neg-raising verbs in embedded contexts. The presuppositional theory makes reasonable predictions for embedding unter quantifiers, but is faced with the fact that neg-raising verbs don't otherwise look too much like presupposition triggers in terms of their projection behaviour from conditional antecedents and questions and the reactions that a supposed presupposition failure naturally elicits. The homogeneity theory faces none of these problems, but it must be noted that there are some points of divergence between neg-raising verbs and other homogeneous constructions. In particular, neg-raising verbs don't display non-maximal readings to any significant extent and show differential behaviour from definite plurals in the antecedent of conditionals and the restrictors of quantifiers. This is a fact that would require additional explanation.

Thus, while the homogeneity theory is certainly promising and at least on par with previous approaches, I cannot claim to have conclusively shown that this must really be the nature of neg-raising.

7 BEYOND THE INDIVIDUAL DOMAIN

This chapter discusses several further constructions that display behaviours that are perhaps to be analysed as homogeneity and non-maximality with respect to a non-individual domain. Conditionals show homogeneity effects and are well-known for permitting exceptions, which is also true for generics. Embedded *wh*-questions likewise display homogeneity, and there is an argument to be made for parallels between *mention some*-readings and non-maximality.

In large part, this chapter has to remain programmatic, since the framework for homogeneity presented in this dissertations does not extend so far as to include them and enable a full analysis. However, it is my hope that it will contain a number of interesting observations and promising directions for future research.

7.1 CONDITIONALS*

7.1.1 *Homogeneity in Conditionals*

It has long been observed that conditionals also seem to have an extension gap, a phenomenon usually called the *conditional excluded middle*. The conditionals in (1) are true if Adam is happy in (almost) all possible futures in which Nina comes.

(1) a. If Nina comes, Adam will be happy.
    b. Adam will be happy if Nina comes.

Their negations, however, require that Adam is unhappy[1] in all possible futures in which Nina comes; it's not sufficient that he is unhappy in only some of them.

(2) a. If Nina comes, Adam won't be happy.
    b. Adam won't be happy if Nina comes.

This suggests the following hypothesis of how homogeneity plays out in conditionals.

(3) If Nina comes, Adam will be happy.
    TRUE iff Adam is happy in all accessible worlds where Nina comes.
    FALSE iff Adam is unhappy in all accessible worlds where Nina comes.
    UNDEFINED otherwise.

* For dicussion on the contents of this section and more, as well as their warm reception, I am very grateful to the audience at the November 2014 meeting of the *What if – Was wäre wenn* research group at the University of Konstanz, particularly Daniel Dohrn, Brian Leahy, Johannes Schmitt, and Wolfgang Spohn.

1 As this manner of speaking will be helpful for disambiguation later on, I am using *unhappy* to mean *not happy* here and in the following.

It may be argued that what is negated in (2) is really the consequent, and not the whole conditional, in which case no assumption of an extension gap is warranted. But it is not at all clear why this should necessarily be so, and, crucially, why no inverse scope reading is available. On the homogeneity-based view, this immediately follows: the *if*-clause is simply scopeless with respect to negation. This is confirmed by sentences in which wide scope of the negation with respect to the conditional is ensured for independent reasons. In (4), the negative indefinite *nobody* binds a pronoun in the antecedent of the conditional, which it can only do if it has scope over the whole conditional.

(4) Nobody will be happy if they get no Christmas present.
$\rightsquigarrow$ Everybody will be unhappy if they get no Christmas present.

The reading that is obtained is what would be expected on the basis of homogeneity. This is analogous to (5).

(5) Nobody found their Easter presents.
$\rightsquigarrow$ Everybody found none of their easter presents.

Similarly, when a conditional is embedded under a negated neg-raising predicate, the negation is seemingly interpreted in the consequent of the conditional.

(6) Agatha doesn't believe that Adam will be happy if Nina comes.
$\rightsquigarrow$ Agatha believes that Adam will be unhappy if Nina comes.

At the same time, however, it can be shown that it doesn't actually take scope there.

(7) Agatha doesn't believe that anybody will be happy if they get no Christmas present.
$\rightsquigarrow$ Agatha believes that everybody will be unhappy if they get no Christmas present.

If the negation were to take scope locally in the consequent, the sentence would not be downward-entailing with respect to the position of *anybody*, which raises questions for its licensing; and furthermore, even if that were somehow solved, the meaning of *anybody* is existential, so that the meaning in (8) would result, which is definitely not what (7) means.

(8) Agatha believes that there is somebody who will be unhappy if they get no Christmas present.

Finally, *no* as an answer behaves also as we would expect it from the perspective of homogeneity: it cannot be used in a situation where the antecedent neither entails nor precludes the consequent. In such a case, the response particle of choice is *well*.

(9) A: If Adam comes, Nina will be happy.
B: No, you're wrong. $\rightsquigarrow$ If Nina comes, Adam will be unhappy.
B': Well, he may be. $\rightsquigarrow$ If Nina comes, Adam may or may not be happy.

This fits well with the view that *if*-clauses have referential status (Bittner 2001, Bhatt & Pancheva 2006), in particular that they are definite descriptions of pluralities of worlds (Schlenker 2004, Klinedinst 2007). The logical form of a conditional would then simply be as in (10).

(10) a. If p, q.
b. $\lambda w.q(\iota w'.R(w,w') \wedge p(w'))$

One is to take the set of all worlds that make the antecedent true and are accessible from the world of evaluation and form the mereological sum of all of these worlds. The consequent, which, being a proposition, is just a predicate of worlds, is then predicated of this plurality of worlds, and since it is a predicate of atomic worlds, this predication is distributive. Consequently, the conditional is true if the consequent is true in all the accessible antecedent worlds, false if it is false in all of them, and otherwise undefined.[2]

7.1.2 *Homogeneity Removal in Conditionals*

As has become apparent, where there is homogeneity, there are usually also expressions like *all* that remove it. Conditionals are a partial exception to this in that there doesn't seem to be a precise analogue of *all*; there is, however, an analogue of *not all*: *not necessarily*.[4] Unlike the plain negation (11b), (11c) is simply true as soon as (11a) is not true — it has no extension gap.

(11) a. If Nina comes, Adam will be happy.
b. If Nina comes, Adam won't be happy.
c. If Nina comes, Adam won't necessarily be happy.

As expected, *not necessarily* goes naturally with a *well*-response to a conditional in a situation in the extension gap. Compare (12) to (13).

(12) Context: *If Nina comes, Adam might be happy or he might not.*
A: If Nina comes, Adam will be happy.
B: Well, not necessarily.

(13) Context: *Adam read some of the books.*
A: Adam read these books.
B: Well, not all of them.

2 This, of course, is a static, propositional meaning for conditionals, which has been questioned on both linguistic and philosophical grounds.[3] Recent dynamic, non-propositional analyses (e. g. Schulz 2007, Schmitt 2012a, Aher 2015) usually incorporate the conditional excluded middle, but frequently in the form of explicitly stated falsity conditions. The only exception to this is Schmitt's, where the conditional excluded middle is a consequence of the way dynamic negation is defined. Such analyses are at this point incommensurable with the homogeneity perspective.

4 Philosophers seem to be able to use *necessarily* without negation, but for ordinary speakers, this is somewhat unidiomatic.

What we furthermore find are analogues of various other adverbial quantifiers: *probably* and *possibly/might*, for example, can be seen as corresponding to *mostly* and *partly* in the individual domain.

7.1.3 *Exceptions and Sobel Sequences*

It is well-known that conditionals are tolerant to exceptions in that in evaluating them, one can leave aside some very far-fetched possibilities.

(14) If Nina comes, Adam will be happy.

Since Lewis 1973, certain troublesome sequences of conditionals are known as *Sobel sequences*. They are little discourses consisting of two consecutive conditionals of the general form given in (15).

(15) a. If p, q.
 b. Of course, if p and r, then not q.

(16) is an example of such a Sobel sequence.[5]

(16) a. If Nina comes to the party, Adam will be happy.
 b. Of course, if Ginger comes too, Adam will be unhappy (because he always flirts with Nina).

The problem that Sobel sequences pose is that if they are monotonic with respect to their antecedents, the two conditionals should be inconsistent: (16a) says that in all accessible worlds where Nina comes, Adam is happy. (16b) says that in some of those worlds (namely those where Ginger also comes), Adam is unhappy.

However, if the semantics is adjusted so as to be non-monotonic and allow the two conditionals the be consistent, then one is faced with the need to explain the fact that Sobel sequences cannot felicitously be reversed, as pointed out by von Fintel (2001) (who credits Irene Heim with the observation).

(17) a. If Nina and Ginger both come to the party, Adam will be unhappy.
 b. #Of course, / #But if Nina comes, Adam will be happy.

This phenomenon would not be mentioned here if it were not so strongly reminiscent of non-maximality, and indeed I will argue that what is behind it is the non-maximal readings that are expected from a homogeneous construction.

7.1.4 *A Note on the Proper Form of Sobel Sequences*

Many supposed examples of Sobel sequences in the philosophical literature, and the linguistic literature derived from it, are given with *but* instead of *of course* (some also have *but of course*). I content thad, to the extent that such sequences of

5 Lewis's original example, and many following him, are subjunctive conditionals. However, as pointed out by Williams (2008), Sobel sequences with indicative conditionals are perfectly felicitous. In order not to have to deal with the additional complications of counterfactuals, I will only discuss indicative examples except in section 7.1.7.

conditionals with *but* are felicitous at all, they are not actually Sobel sequences, but exemplify a different phenomenon.

(18) ?If NINA comes to the party, Adam will be happy, but if NINA AND GINGER come, he will be unhappy.

In order to have any semblance of felicity, (18) requires a very strong constrastive accent on the constituents *Nina* in the first antecedent and *Nina and Ginger* in the second.[6] This intonation is known to be associated with what may be local exhaustification or perhaps reinterpretation of lexical meaning.

(19) a. If it's WARM, we'll lie out in the sun. But if it's VERY warm, we'll go inside and sit in front of the air-conditioner.
b. If you eat SOME of the cookies, they'll make you happy, but if you eat ALL of them, you'll get sick.

In (19a) (from Geurts 2010: 181), *warm* appears to be reinterpreted as *warm, but not excessively so*; in (19b), *some* comes to mean *some but not too many*. In the same way, *Nina* in (18) would come to mean *only Nina*. Note that these uses also do not require the possibilities of it being very warm, or of you eating all of the cookies, to be particularly unlikely and remote, and the same seems to me to be true for Ginger's coming in (18), which sets it these examples further apart from Sobel sequences.

7.1.5 *Sobel Sequences as Non-Maximality*

It turns out that Sobel sequences are just the conditional version of how exceptions can and cannot be mentioned in the case of definite plurals (cf. section 3.1.2).

(20) a. If Nina comes to the party, Adam will be happy. Of course, if Ginger comes too, Adam will be unhappy (but that's really unlikely).
b. The professors smiled. Of course, Smith didn't (but then he never does).

(21) a. #If Nina and Ginger come, Adam will be unhappy, although / but if Nina comes, he'll be happy.
b. #Smith didn't smile, although / but the professors did.

This makes it appear quite plausible that exception tolerance in conditionals should be analysed in the same terms as non-maximality with definite plurals, which will be further supported by considerations in the sections to follow. I do not have a complete formal modal of the denotation of a conditional and of how it can be used to address an issue, but the theory of non-maximality from chapter 3 can still be applied intuitively. At the core of it lies this generalisation: a sentence can be used as long as it is true enough, where *true enough* means that

6 Arguably, what we are seeing there is a contrastive topic construction, cf. Büring to appear for an excellent exposition of this phenomenon.

(i) it is not false and (ii) the actual situation is, for current purposes, equivalent to a situation where the sentence is literally true.

Assume the purposes of the conversation are to decide what actions to take with respect to inviting people to the party. Then it is predicted that in the context in (22), the conditional is interpreted non-maximally.

(22) Context: *We have no way of reaching Ginger, but it's unlikely that he'll come on his own.*
Optimal Action: *Invite Nina and do nothing about Ginger.*
If Nina comes, Adam will be happy. (So we should invite her.)

If the conditional were literally true, the optimal action would of course be to invite Nina. In the supposed scenario, the conditional is not literally true, but undefined, since in almost all of the worlds where Nina comes, Adam will be happy, but in those unlikely worlds where Ginger comes as well, he won't be. But the optimal action is still the same: since we have no way of influencing Ginger, we just have to live with the unlikely possibility that he'll spoil Adam's mood. The optimal action is the same in the actual situation as it would be if the conditional were literally true, and so the sentence can be used non-maximally.

But now consider the slightly different context in (23).

(23) Context: *We can reach Ginger and tell him whether Nina is coming. If we lie and tell him she's not coming, he won't come, either.*
Optimal Action: *Invite Nina and tell Ginger she won't be coming.*
#If Nina comes, Adam will be happy. (So we should invite her.)

If the conditional were literally true, so that Ginger's coming would have no ill effect on Adam, then the optimal action would be to invite Nina, and perhaps even Ginger as well. But in fact, Ginger could spoil the party for Adam, and we have an influence on whether or not he will comes. We should therefore make use of this, and so the optimal action involves making sure that Ginger won't come. So the optimal action in the actual situation is not the same as it would be if the conditional were literally true and the sentence cannot be used in an exception-tolerant manner in this context. This strikes me as quite plausible, although of course rigorously testable predictions are not easy to identify.

7.1.6 *Sobel Sequences and Reference Restriction*

All attempts at making sense of Sobel sequences and their irreversibility that I am aware of have one thing in common: in the first conditional, those worlds that make the second conditional true are somehow excluded as as non-salient or outside of the domain of modal quantification (Lewis 1973, von Fintel 2001, Gillies 2007, Williams 2008, Moss 2012). (24a) is understood to mean that in all the currently salient worlds where Nina comes, Adam is happy. If Ginger's coming is a sufficiently remote possibility, then there will simply be no world among these where Ginger also comes. But (24b) talks about worlds where Gingers comes, and so such worlds have to be considered, however remote they are. Crucially, however, the worlds that are quantified over in the second conditional are not a proper subset of the worlds that are talked about in the first.

(24) a. If Nina comes, Adam will be happy.
b. Of course, if Ginger comes, too, Adam will be unhappy.

The exceptional worlds are only raised to attention and taken into account once the second conditional is uttered. This is, then, the basis for their irreversibility: if the second conditional is uttered first, then these worlds, having just been mentioned, cannot be left out from the domain of the second conditional, and so the first conditional can no longer be regarded as true.

7.1.6.1 *Compositionality Troubles with* necessarily *and Possibility Modals*

Schlenker's theory that *if*-clauses denote pluralities of worlds elegantly explains why conditionals show homogeneity and also provides a nice and simple compositional semantics. Schlenker himself suggests exception tolerance and Sobel sequences are to be explained by the fact that the *if*-clause doesn't denote the sum of all antecedent worlds, but picks out only those above a certain threshold of salience.

A problem then arises which was already noted by Schlenker himself: how does the addition of *necessarily* in the consequent influence the reference of the *if*-clause so as to remove the salience effects? The very worlds that are ignored in (25a) — those where Ginger comes, too, and Adam is unhappy — are the ones that make (25b) true.

(25) a. If Nina comes, Adam will be happy.
b. If Nina comes, Adam won't necessarily be happy.

The same point can be made with a possibility modal in the consequent: a speaker who is disposed to accept (26a) by way of ignoring some exceptions need not be disposed to reject (26b). In order to judge the latter sentence, even remote possibilities have to be taken into account that can be ignored for the purposes of (26a). But how can the addition of a possibility modal in the consequent change the reference of the *if*-clause?

(26) a. If Nina comes, Adam will be happy.
b. If Nina comes, Adam might still be unhappy.

The only way for the traditional view of Sobel sequences to deal with this problem is to revert to the tradition following Kratzer 1986 in which a silent modal is postulated in the consequent of an unmarked conditional (i. e. one that one does not contain an overt modal operator). It would be this silent modal which would both narrow the domain to those worlds that are most salient and furthermore introduce homogeneity. The modal would take as its argument two propositions, the consequent and the antecedent. The contextual variable *salient* is true of a world w and proposition p if w is among the most salient p-worlds.

(27) $[\![\textsc{mod}]\!] = \lambda p.\lambda q.\lambda w.p(\iota u.salient(u, \lambda v.R(w,v) \wedge q(v)))$

Now the *if*-clause denotes just a proposition, not a definite description, but the overall logical form of the conditional still turns out in a way that is compatible with explaining the homogeneity of conditionals as due to distributive plural predication in the world domain.

Necessarily would now completely replace the silent modal, simply being a universal quantifier. It doesn't actually remove homogeneity and exception tolerance, it just doesn't add them in the first place.

(28) $[\![\text{necessarily}]\!] = \lambda p.\lambda q.\lambda w.\forall w' : (R(w, w') \wedge q(w')) \rightarrow p(w')$

The assumption of such a silent modal is, of course, as undesirable as it would be to postulate that every plural quantification is mediated by a silent quantifier (cf. section 1.6.6), as it has never been observed anywhere.[7] Furthermore, this particular silent modal seems rather more complex than the overt ones, making the most unmarked conditionals the ones with the most complicated semantics.

The analysis in terms of non-maximality has no problem with this: it can make use of Schlenker's simple compositional semantics without postulating a dubious silent modal to explain homogeneity, and *necessarily* just removes the homogeneity of the plural predication in the same way that *all* does for individuals. The disappearance of exception tolerance is then just a pragmatic epiphenomenon.

7.1.6.2 *Diagnostics of Reference Restriction*

In section 3.1.5, I have argued that non-maximality with definite plurals is not due to a restriction of the reference of those descriptions. One piece of evidence is that exceptions ignored by way of non-maximality can always be felicitously brought up as a challenge, whereas individuals that were properly outside of the domain of a definite description cannot.

(29) A: The professors smiled.
B: Well, actually, Smith didn't.
B′: Well, yeah, but you know, he NEVER does.

(30) *Uttered at the ENS in Paris.*
A: The students are happy.
B: #Well, actually, the students at the Sorbonne aren't.
A′: What? I wasn't talking about THEM.

The second argument was that with conjoined predicates, it is possible to understand the first predication non-maximally, but the second predication maximally. Even if Prof. Smith is irrelevant as an exception to smiling, this doesn't mean that he is also irrelevant as an exception to leaving. A maximal reading can furthermore be enforced by just adding adverbial *all* in the second conjunct.

(31) *All the professors except Smith smiled and then left, leaving Smith behind.*
#The professors smiled and then (all) left the room.

7 Their ability to do without a silent modal is one of the virtues of dynamic theories of conditionals following Veltman 1996. Schlenker's is the only static approach I am aware of that avoids it.

Finally, a closely related argument can be made with anaphoric pronouns: those have maximal reference even when the predication in which the referent was first introduced is understood non-maximally. (32) doesn't mean that only those professors who smiled left the room.

(32) The professors smiled. Then *they* (all) stood up and left the room.

It was further observed that these diagnostics distinguish non-maximality from salience-based reference restriction in definite plurals.

To the extent that the these tests are at all applicable to conditionals, they pattern with definite plurals in this respect. First, exceptional possibilities that are glossed over can always be felicitously raised, forcing the speaker who asserted the original conditional to justify the omission.

(33) A: If Nina comes, Adam will be happy.
B: Well, actually, if Ginger comes, too, Adam won't be happy.
A': Well, yeah, but come on, how likely is that.

This is different from world that are *bona fide* outside of the domain. With an indicative conditional, such worlds are all those that are epistemically impossible.

(34) *It's known that Ginger won't come.*
A: If Nina comes, Adam will be pleased.
B: #Well, actually, if Ginger comes too, Adam won't be happy.
B: #Well, actually, if Ginger came too, Adam wouldn't be happy.

Worlds that are ignored because they are impossible clearly have a very different status from worlds that are ignored as irrelevant exceptions. The fact that existing theories assign them the same technical status is therefore troublesome: both kinds of worlds are just not among those that are being quantified over. Now it could be suggested that there are still different reasons for why they are excluded from the domain of quantification, and that bringing them up, as in (33), is just a way of saying that the reason wasn't good enough and they should actually be talked about. This defence hinges on how natural it is to use *well, actually* in bringing up additional relevant individuals which were clearly not part of the domain of quantification at first. It is not clear to me that such things actually occur in discourses and I have not been able to find a natural example that would be analogous to the situation with conditionals.

The other two diagnostics are, unfortunately, not very well applicable to conditionals. The reason why an example like (31) works is that with definite plurals, the predicate has a huge influence on who can be ignored as an exception for what reasons. The same factor that makes somebody's failure to smile irrelevant — for example, that they never smile anyway and so it doesn't mean much — does nothing to make their failure to leave the room irrelevant as well. In conditionals, on the other hand, what makes a possibility irrelevant is usually its far-fetchedness or the extent to which we have an influence on its coming about. These are things that do not depend on the consequent conditional in any way. Even enforcing maximality with *necessarily* does not work particularly well, especially in light of the fact that this adverb must be negated and thereby turns into an existential.

The trouble with (35) is the status of the worlds that are responsible for *if p, then not necessarily r* being true. Either these worlds are exceptions to *if p, then q*, but then it is unintuitive to ignore them when assessing the first consequent. Or these worlds are not exceptions to *if p, then q*, but in that case, they are not ignored anyway and the maximising effect of *necessarily* is invisible.

(35) If *p*, then *q* and/but not necessarily *r*.

To the very limited extent that *necessarily* is acceptable without negation, a semblance of the original argument can be made, although unfortunately the judgment is still not nearly as clear as with definite plurals. (36) would permit for Ginger's coming to prevent Nina's presence from making Adam happy, but even Ginger couldn't disrupt the relationship between Nina's coming and Peter's unhappiness.

(36) ?If Nina comes, Adam will be pleased and Peter will necessarily be annoyed.

The version of the argument with anaphoric pronouns faces the additional problem that world pronouns don't seem to be able to pick up just the antecedent worlds of a preceding conditional. *Then* in (37) would seem to always refer to the worlds in which Nina comes and Adam is happy, not just to the worlds in which Nina comes, even if the relationship between the two is qualified as not being entirely strict.

(37) If Nina comes, Adam will (likely) be pleased, and then Agatha will be bored.

The intended reading can be obtained with modal subordination (Roberts 1989). In any case, however, the argument would suffer from the same defects as the above version with conjoined consequents.

(38) If Nina comes, Adam will likely be pleased. Agatha will be bored.

The diagnostics for reference restriction that were very successful in the case of definite plurals cannot be said to yield a strong argument in the case of conditionals, but what one observes is at least perfectly compatible with the non-maximality analysis.

This concludes the basic argument intended to establish that the homogeneity- and non-maximality-based approach to conditionals is promising. In the following section, I will point out some problems that arise once counterfactuals are taken into account.

7.1.7 *The Problem of Counterfactuals*

So far I have been discussing only indicative conditionals. Counterfactuals carry with them a considerable complication: Sobel sequences would presumably have been the only argument for a non-monotonic semantics for indicative conditionals, but for counterfactuals, there are independent reasons to think that they are non-monotonic with respect to their antecedent.

The intuition has always been that when we evaluate a counterfactual of the form in (39), we look only at those *p*-worlds which, in some sense, deviate minimally from the actual world and check whether *q* is true in those worlds. If a world differs from the actual world beyond what is necessary to ensure that it makes *p* true and is in accordance with general laws, then this world isn't considered for the purposes of the counterfactual. The usual manner of speaking is to say that one looks only at the *closest p-worlds*.

(39) If had been *p*, then would have been *q*.

An excellent example of this is discussed by Schulz (2007).

(40) *Suppose there is a circuit such that the light is on exactly when both switches are in the same position (up or not up). At the moment switch one is down, switch two is up and the lamp is out. Now consider the following conditional:*

If switch one had been up, the lamp would have been on.

The conditional is intuitively true in the situation described, even though the lamp is not on in all possible situations where switch 1 is up: in those where switch 2 is down, it is off. But in the actual world, switch 2 is up, and in evaluating the conditional, we look only at those worlds which deviate from the actual world in the minimal way required to make the antecedent true. Those are the worlds where switch 2 is still up (which doesn't touch the truth of the antecedent) and switch 1 is up as well. What is important is that after making the minimal adjustments requires to make the antecedent true, we then assume that the counterfactual worlds we are looking at follow the same causal laws as the actual world, so we follow the arrow of causality to see what else is the case in those worlds (which possibly isn't the case in the actual world). In this case, the causal laws entail that the lamp is one. For a very detailed investigation of this notion of minimal deviation and, in particular, its connection to causal reasoning, see Schulz 2007.

Now it seems clear that counterfactuals must be non-monotonic with respect to their antecedent. In the scenario described in (40), (41a) is true and (41b) is false.

(41) a. If switch one had been up, the lamp would have been on.
b. If switch one had been up and switch two had been down, the lamp would have been on.

And yet, Sobel sequences with counterfactuals have obligatory *of course* and are not easily reversible. Assume that it is known that neither Nina nor Ginger were at the party (but Adam was).

(42) a. If Nina had come to the party, Adam would have been happy.
b. Of course, if Ginger had come too, Adam would have been unhappy.

(43) a. If Nina and Ginger had come, Adam would have been unhappy.

b. #Of course, if Nina had come, Adam would have been happy.

One may suspect that this is because natural language conditionals do not strictly follow the causality-based algorithm described above, where we only trace the arrow of causality in the forward direction: it is known that there is at least some limited amount of causal backtracking in natural language conditionals (e. g. Dehghani et al. 2012), where along with the antecedent, we also accommodate some preconditions that would have given rise to the antecedent being true. And maybe the factors that counterfactually enable Nina's coming might also do the same for Ginger, which would explain why in many contexts, one would probably accept (44).

(44) If Nina had come, maybe Ginger would have come, too.

Strangely, even if it is made explicit that this is not the case, counterfactual Sobel sequences can still not simply be reversed.

(45) a. Ginger was in Paris, so he couldn't have come to the party.
b. But if both Nina and him HAD come, Adam would have been unhappy.
c. #Of course, if Nina had come, Adam would have been happy.

It is, however, possible to reverse counterfactual Sobel sequences of the form *if p, then q; if* $p \wedge r$*, then* $\neg q$ only if they are interrupted by something like *if p, then* $\neg r$:

(46) a. If Nina and Ginger had come, Adam would have been unhappy.
b. Of course, Ginger was in Paris at the time and couldn't have come.
c. So if Nina had come, Adam would have been happy.

I have no explanation to suggest for this confusing situation,[8] but suspect that a way to make sense of it will emerge once a better understanding of how, exactly, human speakers deviate from strict causal reasoning, and of how counterfactual possibility statements are assessed, which at this point we have virtually no theoretical understanding of.

7.1.8 *Conclusion*

Conditionals are known for having a property that looks very much like homogeneity in plural predication. Very roughly speaking, they are true if, assuming the antecedent, the consequent is true; and they are false when, assuming the antecedent, the consequent is false. If the consequent could be either true or false, given the antecedent, depending on further additional circumstances, then they are neither true nor false. At the same time, they are known for being tolerant of some exceptions. It can be shown that these phenomena are very much parallel to homogeneity and non-maximality in plural predication, providing support for the idea that *if*-clauses are referential and denote pluralities of possible worlds, and offering a new explanation for the behaviour of so-called Sobel sequences.

8 Neither does, as far as I am aware, anyone else.

Future research will have to answer the question of of how to explain the puzzling behaviour of counterfactuals in this connection, which I believe to require a deeper understanding of the notion of counterfactual possibility in particular and the way that human speakers reason about it.

7.2 GENERICS

7.2.1 *Homogeneity and Its Removal in Generics*

It has long been noticed that bare plural generics show homogeneity in the same way as definite plurals (Löbner 2000, Cohen 2004, Magri 2012). (47a) is true if pretty much all ordinary dogs are intelligent, but its negation (47b) is true only if pretty much none are.

(47) a. Dogs are intelligent.
b. Dogs aren't intelligent.

That this is not just a matter of some sort of generic quantification taking scope over negation is shown by the fact that it is not possible to assert simultaneously that (47a) is false and that some or many dogs are intelligent (Magri 2012).

(48) #It's false that dogs are intelligent, but some / many of them are.

This looks exactly like homogeneity with definite plurals, and in just the same way, it disappears once *all* is added.

(49) It's false that (all) dogs are (all) intelligent, but some / many of them are.

In addition, *always* can also be used to remove homogeneity in generics.

(50) It's false that dogs are always intelligent, but some / many of them are.

Furthermore, the answers to undefined generic statements look exactly like those to undefined statements with definite plurals.

(51) A: The professors smiled.
B: Well, half of them.
B: Well, actually, Smith didn't.

(52) A: Dogs are intelligent.
B: Well, many of them.
B: Well, actually, some (breeds) aren't.

This strongly suggests that what is seen here is exactly the same thing as homogeneity with definite plurals.

There are, very broadly speaking, two traditions regarding the logical form of generics. One tradition views generics as modal quantificational statements similar to conditionals,[9] so that (53a) has a logical form similar to that of (53b).

(53) a. Dogs are intelligent.
b. If something is a (normal) dog, it is intelligent.

If the connection to conditionals is it taken seriously, then homogeneity is expected, since conditionals are homogeneous, although of course there is the question of how exactly this logical form comes about. The focus in this has generally been on the role of adverbial quantifiers like *always* and *usually*; how *all* and the other adnominal quantifiers in generics are supposed to enter into the compositional semantics in such a way that they end up as quantifiers over situations is quite unclear to me.

The other tradition takes generic statements to be ordinary predicational structures where the bare plural refers to a particular sort of entity, a *kind* (e. g. Carlson 1997, 1980, Liebesman (2011)). Provided that the domain of kinds has a suitable algebraic structure[10] and measure functions defined on it, everything that has been said about homogeneity and quantification in plurals can then be directly applied here.

7.2.2 *Non-Maximality and Two Kinds of Exceptions*

If there is one construction in natural language that is known for looking like a universal that allows some exceptions, it is generics. (54) doesn't say that strictly all birds can fly; rather, it conveys that all stereotypical, normal birds can fly. This exempts very atypical birds such as penguins, and also birds in extraordinary circumstances, such as those with injured and clipped wings.

(54) Birds can fly.

In fact, these exceptions can even make up the majority of instances of the kind when the right predicates are involved. (55), for example, is still appropriate when in fact the majority of individual mosquitoes don't carry the virus.

(55) Mosquitos carry malaria.

Since generics are homogeneous, it is expected, on the view I have presented, that they should allow for exceptions by way of non-maximality. In particular, a bare plural generic of the form in (56) should be usable if at least some X are P and furthermore the situation is, for current purposes, equivalent to one where every single instance of X is P, i. e. the exceptions are irrelevant.

(56) X are P.

9 E. g. Carlson 1989, Krifka et al. 1995, Greenberg 2007, Leslie (2013). I view those who treat generics in terms of non-monotonic logics as essentially also belonging to this tradition: Delgrande 1987, 1988, Morreau 1992, Asher & Morreau 1995.

10 The subkind relation that holds between *penguin* and *bird* will certainly play a role here.

This is plausible enough for the above examples. In the kind of contexts where (54) would normally be uttered, it doesn't really matter that there are a few weird species of birds that can't fly, or that there are certain conditions by which a bird can be afflicted that prevent that individual from flying. Similarly, (55) would be uttered to convey that one should be worried if one has been bitten by a mosquito, or to argue that it is worth considering how to eradicate the species, or something similar, for which purpose it is irrelevant that not all individual mosquitos carry the virus. It does seem, therefore, that at least a lot of the exception tolerance of generics can be explained by non-maximality.

There are, however, also exceptions of a different sort: these are systematic in such a way that at least the theory of non-maximality presented in chapter 3 cannot deal with them.

(57) Mammals suckle their young.

Male mammals obviously do not and cannot suckle their young, and neither do those females who haven't reproduced. Here the exceptions are not somehow atypical or unrepresentative for the kind, nor can it reasonably be said to be irrelevant that the species has males. Rather, the exceptions are such that the predicate is somehow fundamentally inapplicable to them — either because its presupposition that they have offspring is not fulfilled, or (in the case of males) because they are not the right kind of creatures to suckle anyone.

This kind of exceptions cannot be captured as non-maximality on the proposed theory: obviously, (57) cannot be paraphrased as saying that something is the case which is, for current purposes, equivalent to every mammal suckling its young, because a world where literally every mammal suckles its young is basically inconceivable, and anyway we know that we aren't in such a world.

It seems to me that these kinds of exceptions ought to be treated in a different way; they are a feature of what it means for a predicate to apply to a kind, as opposed to an individual. As a further piece of evidence, note that the addition of *all*, which ought to remove non-maximality, does not detract from the possibility of exceptions of this kind: (58) is still perfectly true, intuitively because, indeed, absolutely all species of mammals have the property of suckling their young.

(58) All mammals suckle their young.

7.2.3 *Conclusion*

Generic statements with bare plurals show homogeneity effects in just the same way as sentences with definite descriptions, including the possibility of removing homogeneity by adding certain quantification items, in some cases, such as *all*, the very same ones. What is less clear at this point is what the algebra is with respect to which the homogeneity constraint holds here.

Being possibly the most famous type of exception-tolerant statements, it is tempting to think that this exception tolerance is the consequence of the potential for non-maximal readings, which the theory from chapter 3 predicts for sentences with a homogeneity-induced extension gap. Indeed, it can be shown that some exceptions to generic statements behave very much like those in non-maximal readings of plural predication. However, there is also a second type of exceptions, which behave differently and which the theory of non-maximality cannot explain at all. These are arguably rooted in something different, such as what it means for a predicate to apply to a kind. Future research will hopefully further illuminate the picture.

7.3 QUESTIONS

7.3.1 *Homogeneity with Embedded Questions*[11]

A sentence with an embedded *wh*-question like (59) entails that Agatha knows all the true answers to the question,[12] that is to say, she is required to know of everybody who came that they came.

(59) Agatha knows who came to the party.

This requirement — knowledge of all true answers — is knowns as the *weakly exhaustive* (WE) reading. It is one of three readings under discussion in the literature. The other two have additional requirements that concern the *false* answers to the question. The so-called *intermediate exhaustive* (IE) reading requires that Agatha must not falsely believe an answer to be true that is, in fact, false. On this reading, (59) is true if and only if Agatha knows of everybody who came that they came, and for everybody who didn't come, she either knows that they didn't come or has no opinion about whether they came. The *strongly exhaustive* (SE) reading, finally, requires that Agatha must know of all the false answers that they are false, i. e. that she must have full knowledge of who came and who didn't.[13]

(60) WEAKLY EXHAUSTIVE READING
Agatha knows all the true answers.

INTERMEDIATE EXHAUSTIVE READING
Agatha knows all the true answers and doesn't wrongly believe a false answer to be true.

STRONGLY EXHAUSTIVE READING
Agatha knows all the true answers, and knows that all the false answers are false.

The WE reading is a common core of all three of these readings. If its requirement is not fulfilled, then none of the three readings can be true. And yet we are not at all inclined to call the *negation* of (59) true in a situation where Agatha is fully

11 The fact that embedded questions show something that is very much parallel to homogeneity with definite plurals has been noticed by a number of people, including Benjamin George and Benjamin Spector (p. c.), but I am not aware of any discussion of it in published work, apart from the recent Xiang 2014.

12 An answers to the question *Who came?* is a sentence of the form *x came*; for example, *Adam came*. I will also refer to the propositions denoted by such sentences as answers.

13 In this connection, see e. g. George 2011, Klinedinst & Rothschild 2011, Égré & Spector 2014, and references therein.

informed about who was at the party except that she wrongly believes Miles, who was, in fact there, to have been absent.

(61) Agatha doesn't know who came to the party.

It's not entirely clear what exactly (61) requires to be true, but two intuitive candidates are in (62).

(62) a. For nobody who actually came does Agatha know that they came.
b. Agatha doesn't know of anybody whether they came.

In any case, there is an obvious gap between the extensions of the positive sentence (59) and its negation, and the *all-or-none* flavour of the phenomenon is strongly reminiscent of homogeneity. I suggest that it is, in fact, the very phenomenon. Ignoring, for simplicity, the role of false answers, the picture looks like this.

(63) Agatha knows *Q*.
TRUE iff Agatha knows all true answers to *Q*.
FALSE iff Agatha knows no true answer to *Q*.
UNDEFINED otherwise.

At this point it is unclear what the algebra is that underlies homogeneity with embedded questions. A starting point may be provided by Lahiri 2000, but the role of false answers and the data to be presented in the next section significantly complicate matters. The discussion will therefore proceed on an informal level, and I will continue to simplistically base it on the WE-reading and ignore the sensitivity to false answers. Nevertheless a substantive and interesting picture will emerge.

7.3.2 *Homogeneity Removers in Questions*

Adverbial quantification with respect to embedded questions is a well-known phenomenon (cf. Lahiri 2000). Just as with individuals, we find, for example, *mostly* and *partly* performing this function.

(64) a. Agatha mostly knows who came to the party.
b. Agatha partly knows who came to the party.

Curiously, there does not seem to be an expression that clearly corresponds to *all* for embedded questions in English. What there is is *exactly*, which performs a very similar function. Without delving into the details of false answer sensitivity, it is clear that at least (65) is simply false if there was one guest of whose attendance Agatha is unaware.

(65) Agatha knows exactly who came to the party.

Intuitively, *exactly* doesn't form a scale with *mostly* and *partly*, as *all* does in the individual domain, but with *roughly*. The meaning of the latter is quite unclear to me, but it may be that *roughly* and *exactly* are to be analysed in slightly different terms than the other adverbial quantifiers. Perhaps, then, there is no exact analogue of *all* for embedded questions in English, but there needn't be because *exactly* fills the same role, even if it achieves the semantic effect in a slightly different manner.

A more interesting language to look at in this connection is German. It has a family of elements that are morphologically related to *all* which remove homogeneity in *wh*-questions. These are not found in the matrix clause, but occupy the position of an adverbial quantifier in the embedded question as if they associated with the *wh*-constituent.[14]

(66) Agatha weiß, wer aller auf der Feier war.
Agatha knows who ALL *at the party was*
∼ 'Agatha knows who was at the party.'

The normal position of these elements is, indeed, that of an adverbial quantifier that associates with the *wh*-constituent, but is not directly attached to it. This can be seen when the *wh*-word comes from an object position. The adverbial *alle* in (67b), which associates with the object,[15] is found in the same position as *aller* in the embedded question in (67a).

(67) a. Agatha weiß, wen Nina aller geküsst hat.
Agatha knows who Nina ALL *kissed has*
∼ 'Agatha knows whom Nina kissed.'
b. Die Buben hat Nina alle geküsst.
the boys has Nina ALL *kissed*
'Nina kissed all the boys.'

Having the homogeneity remover attach directly to the *wh*-constituent is marginally possible.[16]

(68) ??Agatha weiß, wen aller Nina geküsst hat.
Agatha knows who ALL *Nina kissed*

Depending on the *wh*-constituent, different specific items are used. They are given in Table 7.1.

Their effect is exactly analogous to that of *all* in predications involving plural individuals. The addition of *aller* in (66) doesn't seem to change the truth conditions, but does extend the falsity conditions to cover the extension gap.[17] Its negation (69) is true as soon as there is one person such that they were at the party and Agatha doesn't know it.

14 Yimei Xiang (p. c.) has pointed out to me that Chinese has an item that seems to function the same way.

15 Unlike its English counterpart, German adverbial *all* can do this, although it occupies a different syntactic position in this case.

16 This is something one would expect to be subject to dialectal and idiolectal variation.

17 It is not intuitively clear to me how, if at all, *aller* influences the role of false answers. This is something that should be explored experimentally.

wer 'who'	*aller*
was 'who'	*alles*
wo 'where'	*überall* 'everywhere'
wann 'when'	*immer* 'always'
welche 'which.PL'	*alle*

Table 7.1: Homogeneity Removers in German Questions

(69) Agatha weiß nicht, wer aller auf der Feier war.
Agatha knows not who ALL *at the party was*
∼ 'Agatha doesn't know exactly who was at the party.'

How these elements come to have this effect is, of course, a mystery, especially in light of their position in the embedded clause, and raises questions for the compositional semantics of *wh*-questions. It should be noted that they are not restricted to embedded questions. Employed in direct questions, they seem to somehow convey that the speaker expects an exhaustive answer.

(70) Wer war aller auf der Feier?
who was ALL *at the party*
∼ 'Who was at the party?'

7.3.3 *Non-Maximality and* mention some-*Readings*

If what has been said in chapter 3 is on the right rack, then sentences with embedded questions should, in virtue of their homogeneity-based extension gap, in principle allow for non-maximal readings. (71) should, for example, allow for some gaps in Agatha's knowledge as long as those gaps are irrelevant for current purposes.

(71) Agatha knows who was at the party.

The corresponding German sentence with *aller*, however, should not. This seems intuitively right, but if at all existent, the allowance for exceptions even in (71) is very small. This, however, may be simply because it is difficult to think of an appropriate context where this particular question would have a strongly non-maximal reading.

There is, however, a famously week reading of *wh*-questions, the so-called *mention-some* (MS) reading. (72) is typically understood as conveying that Mary knows some place where one can buy cheese; she doesn't need to know of all such venues.

(72) Mary knows where we can buy cheese.

This can be nicely explained as a non-maximal reading. If what we want is to find a place to buy cheese, then we don't need to know all such places; we merely need to learn one. Thus, if Mary knows one such place, then that's just as good as knowing all of them. No separate semantic reading needs to be derived for the *wh*-question.[18]

On this approach, there is no categorial distinction between MS-readings, which require only one answer to be known (or whatever the embedding verb is), and *mention-all* (MA) readings, which require all answers to be known. The contextual requirements could be much more fine-grained. Imagine I wanted to form a general picture of what sort of crowd was at the party. It is unlikely that learning a single guest's name is enough, but at the same time, I also don't need to know everybody who was there; perhaps not even close to everybody. Some intermediate number of names would suffice. Assume, for example, that Miles, Nina, and Adam are a notorious trio. Each of them alone is harmless, so hearing that Miles was there doesn't allow me to infer much about the character of the event. Learning that all three of them were there, however, does, regardless of who else attended. In such a case, (73) is predicted to be understood as neither universal nor existential. If the trio was there, for example, it could be made true by Agatha telling me that they were, while mentioning nobody else. But it could not be made true by her telling me only that Miles was there. This prediction strikes me as correct.

(73) Agatha told me who was at the party.

The particular perspective I am advocating, on which non-maximality is linked to homogeneity, entails a further prediction: that homogeneity-removers make MS (or intermediate) readings impossible. This, too, turns out to be true. (74a) does require Mary to know all the places of purveyance where the vending of some cheesy comestibles can be negotiated; and (74b) conveys that Agatha listed all the guests.

(74) a. Maria weiß, wo man überall Käse kaufen kann.
Mary knows where one everywhere cheese can buy
'Mary knows all the places where one can buys cheese.'
b. Agathe hat mir gesagt, wer aller auf der Feier war.
Agatha has me said who all at the party was
'Agatha told me of everybody who was at the party.'

MS-readings occur also in direct questions, in that for some questions, no complete answer is expected and any answer given does not trigger an exhaustiveness implicature.

(75) A: Where can I buy some cheese?
B: You can buy cheese at the *Old Cheese Emporium*.[19]
$\not\rightsquigarrow$ You can't buy cheese anywhere else.

18 This reasoning is in the same spirit as van Rooij's (2003) analysis of MS-readings. For him, the operators involved in deriving the meaning of a question are sensitive to a decision problem. What counts as a full answer to the question is any answer that resolves this decision problem. MS-readings arise in those cases where even a single answer is already good enough.

19 Never mind that this is false.

This is in contrast to MA-questions, where answers do generally carry such an implicature.

(76) A: Which books did you read in the last five months?
B. *Decline and Fall*, *Vile Bodies*, and *Brideshead Revisited*.
$\rightsquigarrow$ I didn't read any other books.

Importantly, homogeneity removers in direct questions make MS-reading impossible and make it clear that the speaker wishes to know all true answers, and not just some.

(77) A: Wo kann man überall Käse kaufen?
where can one everywhere cheese buy
'What are all the places where one can buy cheese?'
B: Im *Old Cheese Emporium* und beim *Urbanek*.[20]
$\rightsquigarrow$ One can't buy cheese anywhere else.

The theory of non-maximality developed in chapter 3 is, of course, not applicable to direct questions in its present form. It makes use of pragmatic principles that apply only to (trivalent) propositions. At this point, it is not even clear what kind of object a regular *wh*-question, as opposed to one with a homogeneity remover, denotes, but one may hope that a question-analogue of the theory in chapter 3 can be found to explain MS-readings and their unavailability with homogeneity-removers in direct questions. This seems a formidable task which I will leave for future research.

7.3.4 *Experiments: Xiang 2014*

Xiang (2014) has investigated experimentally how speakers judge the correctness of a sentence with an embedded question in certain configurations. I aim to how the picture that emerges can, if not be predicted, at least be made sense of from the perspective I have presented in the preceding sections.

Xiang tested the two verbs *know* and *tell* with both a MA-question and a MS-question. Furthermore, the embedding verb could either be unnegated or negated, resulting in a total of eight conditions. The context was always of the same shape: the embedding predicate held of two of the three true answers to the question. Schematic examples of items are given below in Figure 7.1 (from Xiang 2014, typography slightly adapted).[21]

Xiang's findings are summarised in Figure 7.2. Consider first the results for positive sentences. Here, the picture for *know* is quite clear and conforms perfectly to expectations. In the case of *tell*, things are much more muddled. It is not clear exactly why *tell* would seem to make subjects uncertain about whether a MA- or a MS-reading of the embedded question is intended. However, the story I have presented at least leaves room for this, since the pragmatic principles governing

20 For some reason, it sounds strange to name just a single place without using *only*. *Überall* in the question seems to raise an expectation that there is more than one true answer.

21 Xiang refers to MA-questions as *exhaustive* (EXH) questions here.

- *Context*: Alice, Bill and Celine are the only three people who passed the math exam. John [knows / told Mary] that Alice and Bill passed the math exam, but he [doesn't know / didn't tell her] that Celine also did.
- *Mary says*: "John [knows / told me] who passed the math exam."
- Is Mary right or wrong? ◯ Right. ◯ Wrong.

(a) MA-questions

- *Context*: There are three places in total selling pre-paid envelopes near the train station, including USPS, UPS, and FedEx. Marek [knows that he / told Mary that she] can buy pre-paid envelopes at USPS and UPS, but he [doesn't know / didn't tell her] that [he / she] can also do that at FedEx.
- *Mary says*: "Marek [knows / told me] where to buy pre-paid envelopes."
- Is Mary right or wrong? ◯ Right. ◯ Wrong.

(b) MS-questions

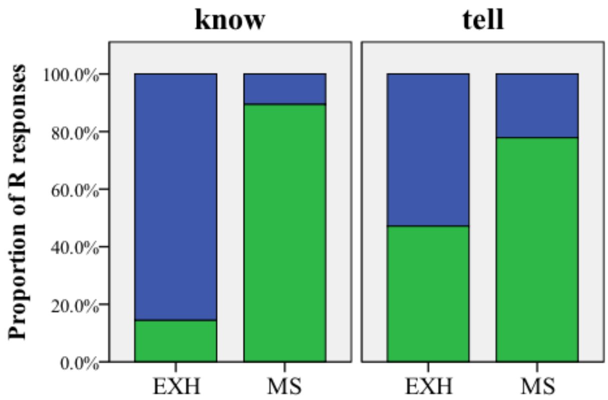
Figure 7.1: Items from Xiang 2014

non-maximal readings are applied on the level of the whole sentence and will therefore take into account the embedding verb.

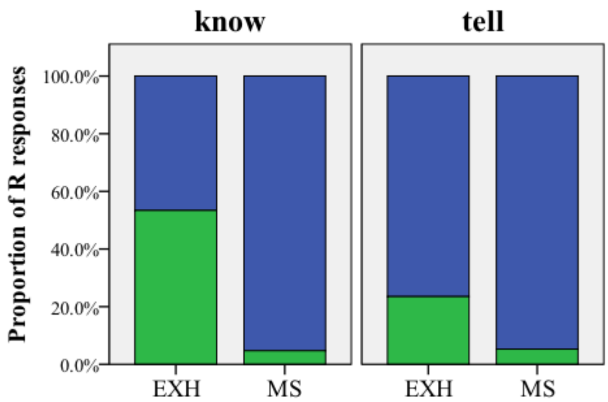
Let us now turn to the negated sentences. From the perspective of homogeneity alone, the high percentage of affirmative answers for *know* with MA-questions is unexpected, but once non-maximality is taken into account, it can intuitively be made sense of. What is required for (78) to be judged (non-maximally) true is that something is the case which is, for current purposes, equivalent to John knowing nothing about who passed. It is thinkable that some subjects would accommodate a context where all that matters is whether John is fully informed or not. In that case, being partly informed is as bad as having no clue at all, and so (78) would be judged true even if John knows about some people who passed.

(78) John doesn't know who passed the math exam.

This seems intuitively quite plausible for *know*. It is perhaps quite often the case that the difference between complete and incomplete information is what is of primary importance. In the case of *tell*, subjects apparently feel less inclined to accommodate such a context. Note that this is what, given the picture I have presented, one expects in light of the data for positive sentences. To the extent that subjects are inclined to judge a partial answer as, for current purposes, equivalent to a complete answer, they should be disinclined to judge it equivalent to no answer. Thus, verbs which show a greater degree of non-maximality in the positive case should show less non-maximality in negative sentences. This is, indeed, what we find with *know* and *tell*.



(a) Positive sentences



(b) Negated sentences

Figure 7.2: Results from Xiang 2014

The extreme of this tendency is, of course, found with MS-questions. If giving some answers is as good as giving all answers, then it cannot be that giving some answer is as bad as giving no answer; or else the question would have to be entirely irrelevant. This explains the categorical rejection of the sentences in the negated conditions with MS-questions.

7.3.5 *Conclusion*

In this section, I have discussed the idea that embedded questions, too, are subject to homogeneity: a sentence with an embedded question is, very loosely speaking and ignoring the intricacies of question embedding, true if the embedding predicate is true of all the true answers, and false if it is false of all the true answers. It is undefined if the predicate is true of some and false of others. This was recently corroborated by Xiang's (2014) experimental results.

Furthermore, adverbial quantification works with embedded question in a manner very much parallel to its use with definite plurals, and some languages, like German, have lexical items which, when added inside the question, remove homogeneity.

van Rooij's (2003) analysis of *mention some*-readings is closely related in spirit to the theory of non-maximality I have presented in chapter 3, and the general association between homogeneity and non-maximality suggests strongly that *mention some*-readings are, in fact, the reflex of non-maximality in the question domain. This is further corroborated by the fact that they become unavailable once homogeneity removers are added to the question, just like non-maximal readings of definite plurals are prevented by the addition of *all*.

This finding raises interesting perspectives for future research. The fact that readings of embedded questions are a good deal more complicated than just truth of the predicate of all true answers raises the question of what, exactly, the object is that embedded questions denote, and what the algebraic structure is with respect to which question-embedding predicates are homogeneous. Furthermore, if MS-readings are an instance of non-maximality, then it must be defined what, exactly, it *means* for a direct question to be used "non-maximally" and the theory of non-maximality need to be extended so as to be able not only to propositions, but also to questions.

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